Why should irrigated agriculture engage in the conservation of soil biodiversity?

Sébastien Foudi[a*] Yoro Sidibé[b] , Unai Pascual[a,c,], Mette Termansen[d]

Preliminary results

a) Basque Centre for Climate Change (BC3), 48940, Leioa, Spain;

b) World Bank, Washington D.C., USA;

c) Ikerbasque, Basque Foundation for Science, Bilbao 48013, Spain;

d) Department of Food and Resource Economics, University of Copenhagen, Denmark;

**) Corresponding author: sebastien.foudi@bc3research.org. Basque Centre for Climate Change (BC3), Scientific Campus of UPV/EHU; Sede Building 1; Barrio Sarriena, s/n 48940, Leioa, Spain.*

Abstract:

Irrigation water use is expected to be constrained as drought frequency increases with climate change. We present a bioeconomic model that illustrates the role of soil biodiversity in agroecosystems and for irrigated agriculture. Soil biodiversity provides an insurance value to irrigating farmers as it enables to transfer water over time and stabilize production. Our results show that different thresholds of risk and inputs costs determine the choice over different farming strategies: irrigated or rainfed and using soil biodiversity conservation practices or not. We show that the optimal levels of soil biodiversity conservation and irrigation water depend on a combination of key hydrological and agronomic factors and depend on economic factors only for a certain value of their costs ratio. The sensitivity of irrigating farmers to price-based regulation is then determined by the ratio of inputs costs, which calls into question the use of prices relative to quota policy instruments to manage both soil biodiversity and water.

Keywords: ecosystem services; bioeconomic modelling; risk and uncertainty; water demand management; agroecology

JEL code: Q1; Q15; Q18; Q57; D81

1 Introduction

Increased crop production for food security has mainly relied on the use of agrochemicals and irrigation together with land expansion. However, the pressure on the ecosystems harms nature contributions to people. In a climate change context, water availability is likely to be more uncertain and more variable (Milly et al., 2008); drought risk may increase in regions such as those under Mediterranean climate and make farmers more vulnerable (Esteve et al., 2015). In response farmers can adopt practices that improve water use efficiency (Hatfield et al., 2001) or more generally agroecological practices (Wezel et al., 2014), those practices that integrate the long-term protection of natural resources into agricultural production for food, biomass or fiber.

Soil biodiversity plays a crucial role in the delivery and regulation of ecosystem services valuable for society (Bardgett and Putten, 2014; Pascual et al., 2015; Wall et al., 2015): soil fertility, plant growth , human health and the climate. It is involved in multiple ecosystem functions and services such as water cycle (Bardgett et al., 2001) or biogeochemical process in the nutrient cycle (Lubbers et al., 2011; Wagg et al., 2014). It has complex interactions with aboveground species (De Deyn and Van der Putten, 2005; Eisenhauer, 2012); plants provide organic carbon and food resources to belowground species while the latter favor nutrient acquisition by the plant. The diversity of species and functional groups (such as decomposers or engineers) influences soil ecosystem services delivery. Spurgeon et al. (2013) show that the abundance and complexity of earthworms and fungal contribute to soil structure stability and water infiltration. Also, both community composition and species richness influence carbon cycling (Nielsen et al., 2011) and global biogeochemical processes (Crowther et al., 2019), which feed the debate about the functional redundancy hypothesis. We thus consider a standard definition of soil biodiversity as "the variation of soil life, from genes to communities and the ecological complexes of which they are part, that is from micro-habitats to landscapes (Turbé et al., 2010).

The capacity of biodiversity to regulate the flow of ecosystem services has been conceptualized as the insurance value of biodiversity (Baumgärtner, 2007). Soil biodiversity has an indirect value as it drives intermediate ecosystem services (Pascual et al., 2015). Appropriate agroecological practices contribute to the conservation and management of soil biodiversity (Lemanceau et al., 2015; Nielsen et al., 2015; Wezel et al., 2014). Farmers can thus optimize the level of soil biodiversity conversation to manage production risk, a level that provides a certain level of production and an insurance against production variability. In the particular case of soil water regulation, Sidibé et al. (2018) show that in rainfed agriculture this level of soil biodiversity which provides an insurance value depends on a combination hydrological, ecological, agronomic and economic factors. The risk aversion of the farmer and the cost of soil biodiversity conservation are of particular importance when determining the optimal level of soil biodiversity for the farming system.

Irrigation is another strategy to complement rainfall. The capacity of irrigation to reduce production risk has been questioned: no consensus has emerged on the capacity of irrigation to reduce production risk in irrigated agriculture (Foudi and Erdlenbruch, 2012; Groom et al., 2008). However, farmers who bear more risk are more likely to adopt water-efficient technologies of irrigation (Koundouri et al., 2006). In a climate change context with more frequent droughts, irrigator water is more likely to face shortages and irrigators would need to also manage that source of uncertainty. Others instruments to manage production risk are crop diversification (Chavas, 2008), crop biodiversity (Di Falco and Chavas, 2009) and crop insurance (Coble and Barnett, 2013).

This paper focuses on the management of water-related production risk with soil biodiversity conservation and irrigation practices. It develops a bioeconomic model which describes basic relations between hydrological, agronomic and ecological principles relating production to soil biodiversity and water cycle. These relations are faced by a risk averse farmer who maximizes an expected utility of profit in such agroecosystem. The famer has to decide the optimal decision regarding soil biodiversity conservation and irrigation level in the context of stochastic rainfall and costly inputs. The paper contributes to understanding farmers' preferences for agricultural production strategies involving the conservation of soil biodiversity and/or irrigation and helps to determine under what conditions the conservation of soil biodiversity benefits irrigated agriculture.

2 A bioeconomic model with soil biodiversity and irrigation

We consider a farming system in which a farmer can decide to grow crops in a rainfed system or using irrigation. We assume soil biodiversity to be a stock of natural capital which regulates soil humidity: it enables to accumulate and store water. Water is an input to the final production of the crop. The cycle of water and the intermediate service of soil biodiversity is similar to that we developed in Sidibé et al. (2018) for a rainfed system. The current model extends to the case where irrigation can be used as a technological input and uses nonlinear production function.

2.1 Water demand, rainfall and soil biodiversity

The rainfall pattern in an agricultural season is described by two stochastic rainfall periods. For each period, a low rainfall π_l characterizing a drought period or a high rainfall, π_h characterizing a humidity period but not floding. These events occur with probability φ_l and $\varphi_h = 1 - \varphi_l$, respectively.

Soil biodiversity participates in the generation of soil, its structure and texture (Altieri, 1999) and as a consequence participates in water retention and infiltration into the soil. Some organisms affect soil permeability, other soil porosity. All in all, all have contributed to the soil's capacity to withhold water. It is assumed that higher diversity of soil biodiversity confers to the soil a higher capacity to store water (Allison, 1973; Bastardie et al., 2005), as represented by equation (1):

$$S_c = L \times [I_b]^{\mu} \tag{1}$$

where S_c is the soil's water storage capacity, L is a proportionality coefficient, I_b is the stock of soil biodiversity and μ is a parameter between 0 and 1. In equation (1), soil biodiversity increases the water storage capacity at a decreasing rate. This accounts for the limited capacity for incremental species to bring additional water holding capacity.

The dynamics of water in the soil is described by a difference equation as a function of the total quantity of water in the porous medium and the intrinsic properties of the medium (Kirkham, 2005; Roscoe, 1968), as per equation (2):

$$V_{t+1} - V_t = -\frac{k}{S_c} V_t \tag{2}$$

where V_t is the quantity of water in the soil at time *t* and k represents some intrinsic properties of the soil. Equation (2) states that a greater volume of water in the soil will lead to a proportionally greater water flow out of the soil. The volume of water at $t+1$ increases as the soil storage capacity, S_c increases. Combined with equation (1) it implies that more biodiverse soils today conserve more water for future use.

The water withdrawn by plants is conditioned by the total quantity of water available. This quantity of water in the soil is an additive function of the water remaining in the soil from period $t-1$ and the water that reaches the soil due to stochastic rainfall level $\tilde{\pi}_t$ at period *t* . When the quantity of water available is larger than the real evapotranspiration, the water is not scarce, the plant can satisfy its water needs so the plant's water withdrawal corresponds to the evapotranspiration, When the water is scarce, the plant is stressed and withdraws the quantity of water available. The amount of water used by plants is then given by:

$$X_t = \begin{cases} ET_t \ if \ ET_t \leq V_{t-1} + \tilde{\pi}_t \\ V_{t-1} + \tilde{\pi}_t \ if \ else \end{cases} \tag{3}$$

where X_t is the uptake of water by the plants at time *t*; ET_t is the rate of real evapotranspiration, and $V_{t-1} + \tilde{\pi}_t$ represents the total amount of water available in the soil.

Combining equations (2) and (3), it results that the quantity of water in the soil at the end of any given period t depends on the level of rainfall and the plants' demand at that period $(\tilde{\pi}_t - X_t)$, the quantity of water remaining from the previous period, V_{t-1}, and the soil biodiversity-water storage function $(1 + \beta)$:

$$V_t = \frac{V_{t-1} + \tilde{\pi}_t - X_t}{(1 + \beta)} \tag{4}$$

where $\beta = \frac{k}{S_c}$. A higher level of soil biodiversity means a lower value of β and implies more water remains in the soil.

The overall plant uptakes during this agricultural season depend on the state of nature imposed by the stochastic rainfall and the water dynamics previously described. Table 1 details these uptakes for the production. In this setting it is assumed that ET_t is higher than the low level of rainfall π_l, as well as lower than the upper level π_h. Hence, $\pi_h = \gamma \pi_l$ and $ET = \alpha \pi_l$ with $1 \leq \alpha \leq \gamma$. The model thus assumes that the water demand of the plant in period t, ET_t, is satisfied with an upper level of rainfall in that period[1]. Also to simplify notations in subsequent equations, we denote $h = 1 + \frac{\gamma - \alpha}{1+\beta}$ where γ is the ratio between the upper and lower level of rainfall, $(\gamma - \alpha)$ is the coefficient of rainfall water remaining in the soil once the plant demand rate α is satisfied, h can thus be interpreted as a fraction of remaining rainfall water stored for the next period by soil biodiversity. When $\gamma = \alpha$, the plant water needs are satisfied by all the rainfall and there is no water to be stored by soil biodiversity.

The capacity of soil to retains water is driven by its saturation point. The soil is said to be saturated when the soil pores are completely filled with water and thus any additional amount of water runs off. This point depends on the pores in the soil. Soil biodiversity, particularly ecosystem engineers such as earthworms, ants, termites or macro vertebrates modify the soil aggregates and greatly contribute to the construction of pores. We defined a level of water saturation in the soil (or saturation point, S_p) relating soil saturation with soil biodiversity:

$$S_p = L' I_b^{\mu'} \tag{5}$$

where L' is a proportionality coefficient and μ' a parameter between 0 and 1. Equation (5) specifies that the saturation level increases with soil biodiversity in a decreasing manner because there are physical limits to increasing saturation level and additional increases are more difficult to achieve.

---

[1] It is also assumed that *ET* is the same for all development phases of the plant.

Table 1: Decomposition of the quantity of water used by plants at each period

States of nature, ***SN***	Rainfall Period 1	Rainfall Period 2	Plant uptake X$_1$ Period 1	Water remaining in the soil between the 2 periods	Plant uptake X$_2$ Period 2	Production
1	π_l	π_l	π_l	0	π_l	$F_1(\pi_l, \pi_l)$
2	π_l	π_h	π_l	0	$ET = \alpha\pi_l$	$F_2(\pi_l, \alpha\pi_l)$
3	π_h	π_h	$ET = \alpha\pi_l$	$\frac{\gamma - \alpha}{1 + \beta}\pi_l$	$ET = \alpha\pi_l$	$F_3(\alpha\pi_l, \alpha\pi_l)$
4	π_h	π_l	$ET = \alpha\pi_l$	$\frac{\gamma - \alpha}{1 + \beta}\pi_l$	$\left(1 + \frac{\gamma - \alpha}{1 + \beta}\right)\pi_l$	$F_4(\alpha\pi_l, h\pi_l)$

2.2 A nonlinear production function

We consider a nonlinear production function. The production function $F(X_1, X_2)$ is zero below a certain level of water withdrawn by the plant. Beyond that level, the production starts sharply increasing (Letey et al., 1990; Mantovani et al., 1995). Such a case represents the case of crops, for which the first quantities of water are used to maintain the vegetative system, including the roots, the stem and the leaves. Only after that, the reproductive system of the plant starts developing in order to give fruits. The marginal value of water is thus zero up to a given level at which the value becomes high.

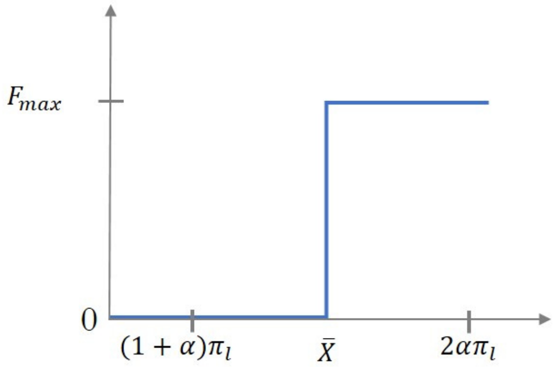


Figure 1: A nonlinear production function

Let us then assume that the marginal productivity of water is zero until the water reaches the amount $X_1 + X_2 = \bar{X}$ and the total production level is equal to F_{max} beyond $\bar{X}$. Let us assume that $(1 + \alpha)\pi_l < \bar{X} < 2\alpha\pi_l$, otherwise the water stress in SN1 would not affect the production level (if $(1 + \alpha)\pi_l < \bar{X}$ is not verified) and the water requirement for a positive production would never be matched (if $\bar{X} < 2\alpha\pi_l$ is not verified) even in the most humid state of the nature SN3 (Table 1).

2.3 A farming system with regulated irrigation

A priori, the farmer may irrigate in both periods. We assume that when the irrigation decision is made in period 1 the farmer has not observed the rainfall levels while when the decision is made for period 2 the rainfall level of period 1 has been observed. In order to choose the amount of irrigation water to use in period 1, we assume the farmer reasons in terms of expected utility by affecting probabilities to the possible events ahead (drought or humidity). As for the irrigation of period 2, the farmer will adapt the level of water according to the level of rainfall observed in the previous period.

We assume that if a drought period occurs, irrigation is banned by the regulator. In case of a dry period 2, the farmer will not be able to irrigate in this period. In the event of a humid period 2, the level of rainfall is such that irrigation will be useless because it has been assumed that plant requirement is satisfied with rainfall of a humid period. It can thus be concluded that the farmer will not irrigate in period 2. This assumption is made to describe a farming system not characterized by permanent water stress in which periods 1 and 2 would represent two water stress periods of different intensity and where the farmer would always irrigate. This assumption enables to isolate the effect of soil biodiversity intermediate service (i.e. the water transfer from period 1 to period 2) on irrigation decision and soil biodiversity conservation level.

3 Optimal choices of irrigation and soil biodiversity conservation

The production function is stochastic as it depends upon the rainfall level $\tilde{\pi}_t$ and its probability of occurrence. The farmer faces thus a production risk[2]. The utility derived from farming will then depend on the output and its distribution. If the farmer is risk averse, a stable produce is preferred to a higher but variable (uncertain) produce. It is then assumed that the farmer is rational and maximizes its expected utility which takes into account risk aversion. We use a linear mean-variance approximation of the market value of production (Levy and Markowitz, 1979; Markowitz, 2014).

---

[2] Price risk is not considered in this study.

In this model the farmer has two decision variables to deal with production risk: the level of soil biodiversity conservation and the level of irrigation water. Those decisions are taken maximizing the expected utility under the assumptions regarding the regulation of irrigation and the production function:

$$\max_{I_b, w_1} E\big(pF(X_1, X_2)\big) - \lambda Var\big(pF(X_1, X_2)\big) - c_b I_b - c_w w_1 \tag{6}$$

under the following constraints:

$$X_1 + X_2 \geq \bar{X} \tag{7}$$

$$X_1 \leq S_p \tag{8}$$

c_w is the unit cost of irrigation water. w_1 is the quantity of irrigation water for period 1. c_b is the marginal cost of soil biodiversity conservation[3]. p is the market price of the crop. S_p is the saturation point defined by equation (5). The constraint (7) means that the water used for the growth of the plant for the two periods should at least equal a given level $\bar{X}$ which enables to reach the maximum production level. The constraint (8) means that the water in the soil should not exceed the saturation point S_p.

Four strategies of soil biodiversity conservation and irrigation can occur: a) use none of the two inputs; b) use only soil biodiversity; c) stop farming the crop and d) use both soil biodiversity and irrigation. We could think of a fifth scenario where the farmer uses only irrigation water but no biodiversity. We will see in section 3.3 that this scenario is not possible and show that the farmer has to forcibly use a certain amount of soil biodiversity along with irrigation. Relying only on irrigation is not an optimal solution because it has been assumed that irrigation is banned in case of a dry period (a drought). During a dry period, the only additional source of water (i.e. non-rainfall water) is the water transferred by soil biodiversity.

3.1 The farmer decides to grow crops without using biodiversity.

This case describes a rainfed intensive farming system in which no land management practices favoring soil biodiversity are implemented, $I_b = 0$ and $w_1 = 0$. Then the farmer's expected utility, R_A writes as:

$$\begin{aligned} R_A &= E\big(pF(X_1, X_2)\big) - \lambda Var\big(pF(X_1, X_2)\big) \\ &= \varphi_h^2 pF_{max}\big[1 - \lambda\big(1 - \varphi_h^2\big)pF_{max}\big] \end{aligned} \tag{9}$$

---

[3] It reflects the efforts made to conserve soil biodiversity through diverse management practices.

We can then define a risk level λ_0 for which the expected utility is zero ($R_A = 0$) and:

$$\lambda_0 = \frac{1}{(1-\varphi_h^2)F_{max}} \tag{10}$$

If the actual risk aversion λ of the farmer is lower than the threshold λ_0 then the farmer may choose to grow crops without resorting to soil biodiversity and irrigation because $R_A > 0$. But if the actual risk aversion is higher than λ_0, the farmer will not grow crops without using biodiversity because $R_A < 0$. In fact, in that case, cultivating without both biodiversity and irrigation would increase the risk the farmer will face because he/she may count only on high levels of rainfall to hope a harvest. A highly risk averse farmer (with $\lambda > \lambda_0$) is thus not likely to choose a rainfed farming without soil biodiversity conservation.

An increase in the drought frequency can make farmers who cultivate in rainfed without any insurance (i.e. with no soil biodiversity or irrigation investments) to revise their farming practices. The threshold λ_0 is increasing in φ_h^2. Therefore, a lower frequency φ_h of high rainfall (equivalently a higher drought frequency) will decrease the risk threshold to $\lambda_0' < \lambda_0$ in a quadratic manner. Those farmers with a modest risk aversion (i.e. $\lambda_0' < \lambda < \lambda_0$) are thus likely to experience a negative expected utility as the threshold moves to λ_0' and prefer another strategy.

3.2 The farmer uses a certain level of soil biodiversity and no irrigation.

In this model soil biodiversity provides an intermediate service of water transfer which contributes to supply plant water needs and consequently to the final production. There is no rational for a farmer to choose a level of soil biodiversity that would lead to a level of agricultural output lower than F_{max}. In fact, doing so, his/her production would be zero in state of nature 4 where soil biodiversity can transfer water from a humid to a dryer period but he/she will be paying the cost of soil biodiversity conservation. In that case, the profit would be lower than in the previous option where he/she uses none of the two inputs. Similarly, because soil biodiversity conservation is costly, a rational farmer will not choose a level of soil biodiversity higher than the level that would lead to the production F_{max}. Therefore, the level of soil biodiversity used will be such that the plant water demand constraint (7) is saturated, enabling to reach the maximum production and satisfies the following equation:

$$X_1 + X_2 = (\alpha + h)\pi_l = \bar{X} \tag{11}$$

$\alpha\pi_l$ being the potential evapotranspiration or plant demand in period 1 and $h\pi_l$ the plant water withdrawal for period 2 in a rainfed system. We recall that $h = 1 + \frac{\gamma-\alpha}{1+\beta}$ with $1 \leq \alpha \leq \gamma$ and $\beta = \frac{k}{LI_b^{\mu}}$, we have therefore:

$$I_b{}^{*} = \left[\frac{k}{L\left(\frac{\gamma-\alpha}{\frac{\bar{X}}{\pi_l}-\alpha-1}-1\right)}\right]^{} \tag{12}$$

The expected utility writes as:

$$\begin{aligned} R_B &= E\big(pF(X_1, X_2)\big) - \lambda Var\big(pF(X_1, X_2)\big) - c_b I_b{}^{*} \\ &= \varphi_h pF_{max}[1 - \lambda\varphi_l pF_{max}] - c_b I_b{}^{*} \end{aligned} \tag{13}$$

We can define a level of risk aversion, λ_{lim}, for which the expected utility is zero ($R_B = 0$) and

$$\lambda_{lim} = \frac{1}{\varphi_l pF_{max}}\left[1 - \frac{c_b I_b{}^{*}}{\varphi_h pF_{max}}\right] \tag{14}$$

The expected utility in a rainfed farming system using soil biodiversity is positive if $\lambda < \lambda_{lim}$. Otherwise it is negative and the farmer might prefer another farming strategy.

A farmer in a rainfed system is likely to invest in soil biodiversity only if the expected utility of this strategy is positive and higher than the expected utility in a strategy without soil biodiversity, namely if $R_B > R_A$ and we have the following condition:

$$R_B - R_A > 0 \; if \quad \lambda > \frac{1}{pF_{max}(\varphi_l^2 - 3\varphi_l + 1)}\left[\frac{c_b I_b{}^{*}}{\varphi_l\varphi_h pF_{max}} - 1\right] = \lambda_L \tag{15}$$

λ_L is the level which makes the farmer indifferent between a rainfed farming with or without soil biodiversity conservation.

The choice between farming in a rainfed system without soil biodiversity (strategy A), farming with soil biodiversity (strategy B) and not farming (strategy C) depends on the drought frequency, i.e. the sign of $(\varphi_l^2 - 3\varphi_l + 1)$, on the cost-price ratio of soil biodiversity, i.e. the sign of $\left[\frac{c_b I_b{}^{*}}{\varphi_l\varphi_h pF_{max}} - 1\right]$, how large the farmer's risk aversion λ is compared to the risk threshold levels λ_{lim} that ensures a non-negative R_B, and compared to the risk threshold λ_0 that ensures a non-negative R_A and to the risk threshold λ_L. We analyse those cases below.

3.2.1 The drought frequency is low

A relatively small probability[4] of low rainfall π_l makes that $\varphi_l^2 - 3\varphi_l + 1 > 0$ in equation (15). Table 2 describes the cases where for a risk aversion motive one strategy is preferred over another. Adding the condition on the cost-price ratio of equation (15) enables to understand which strategy is preferred for which reasons.

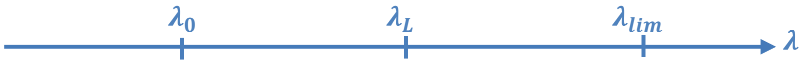
If $c_b I_b{}^* < \varphi_l \varphi_h p F_{max}$, the cost of soil biodiversity is relatively low compared to the output price. Consequently, the value of λ_L is negative in equation (15). As the risk aversion is a positive term, all levels of risk aversion satisfy the condition of equation (15), so $R_B > R_A$. Then all farmers invest in soil biodiversity in this rainfed system except those farmers with a very high risk aversoin, i.e. $\lambda > \lambda_{lim}$, since they would have a negative expected utility, $R_B < 0$. Beyond the threshold λ_{lim}, farmers would stop farming this crop whose water uptake is ET=$\alpha\pi_l$ and would opt for a less water intensive crop or stop farming.

If $c_b I_b{}^* > \varphi_l \varphi_h p F_{max}$ then the cost of biodiversity is relatively high compared to the output price. The condition (15) for strategy B to be preferred to strategy A applies and Table 2 evidences how the level of risk aversion drives the decision. Beyond a certain threshold of risk aversion, $\lambda > \lambda_L$, farmers invest in soil biodiversity to reduce the risk and reach F_{max}. But beyond a second threshold, λ_{lim} they stop farming the crop because their expected utility R_B would be negative. Below the threshold of λ_L but beyond the other threshold λ_0, i.e. in the case where $\lambda_0 < \lambda < \lambda_L$, farmers will stop farming the crop. The farmer's risk aversion is low enough to prefer the strategy without soil biodiversity (A) and face the variability of the rainfed system but it is high enough to not prefer a negative expected utility. Stop farming the crop will thus be preferred. Finally if the farmer's risk aversion is very low $\lambda < \lambda_0 < \lambda_L$, a positive expected utility can be reached so these farmers prefer to grow the crop without soil biodiversity in a rainfed system; the low level of risk aversion makes it unnecessary to resort to a natural insurance such as soil biodiversity in the face of production risk when drought frequency is low and the cost of soil biodiversity conservation high.

---

[4] A numerical resolution gives φ_l< 0.382

Table 2: Summary of decisions intervals driven by risk aversion when the drought frequency is low

	$\lambda < \lambda_L$		$\lambda > \lambda_L$	
	$\lambda < \lambda_0$	$\lambda > \lambda_0$	$\lambda < \lambda_{lim}$	$\lambda > \lambda_{lim}$
Risk aversion				
Expected utility	$R_A > R_B$ $R_A > 0$	$R_A > R_B$ $R_A < 0$	$R_B > R_A$ $R_B > 0$	$R_B > R_A$ $R_B < 0$
Production	$F > 0$	$F = 0$	$F > 0$	$F = 0$
Soil biodiversity	$I_b = 0$	$I_b = 0$	$I_b = I_b^*$	$I_b = 0$
Strategy	Rainfed farming without soil biodiversity (Option A)	Stop farming the crop (Option C)	Rainfed farming using soil biodiversity (Option B)	Stop farming the crop (Option C)

Therefore, when droughts are relatively not frequent, the investment in soil biodiversity requires farmers to experience a certain level of risk aversion. Soil biodiversity provides an insurance to those farmers as it reduces income variability[5]. But a too high risk aversion (i.e. beyond λ_{lim}) would lead those farmers to prefer stop farming the crop as they would receive a negative expected utility. A difference is to note when the cost of biodiversity is relatively high. Farmers should not only have a not too high risk aversion but also a not too small (i.e., not below λ_L) risk aversion. Indeed, below this second threshold λ_L, the high cost of getting a natural insurance does not offset the benefits of being insured. In that case farmers would either farm without soil biodiversity or stop farming.

3.2.2 The drought frequency is high

A relatively high probability of low rainfall π_l makes that $\varphi_l^2 - 3\varphi_l + 1 < 0$ in equation (15). The condition (15) for farmers to invest in soil biodiversity, i.e. $R_B > R_A$ becomes the following:

$$\lambda < \frac{1}{pF_{max}(\varphi_l^2 - 3\varphi_l + 1)}\left[\frac{c_b {I_b}^*}{\varphi_l \varphi_h p F_{max}} - 1\right] = \lambda_L \qquad (16)$$

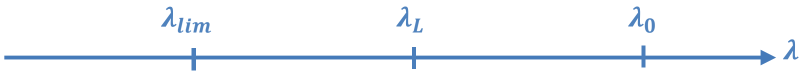
---

[5] The variance in strategy B is lower than in strategy A for this consideration of drought frequency.

If ${c_b I_b}^* > \varphi_l \varphi_h p F_{max}$, the cost of soil biodiversity is high compared to the output price. The condition (16) cannot be satisfied since the risk aversion coefficient λ cannot be negative. The strategy B consisting in farming with soil biodiversity in a rainfed system is never preferred to the strategy A of farming in rainfed without soil biodiversity. Indeed, as the probability of having rainfall is low, the water transfer function of soil biodiversity is less likely to occur. Therefore, investing in soil biodiversity which is costly is not preferred. All types of farmers cultivate in rainfed without soil biodiversity except those farmers with a very high risk aversion ($\lambda > \lambda_0$) which will prefer to stop farming this crop for not receiving a negative expected utility.

If ${c_b I_b}^* < \varphi_l \varphi_h p F_{max}$, the cost of soil biodiversity conservation is lower than the output price and the condition (16) applies. Table 3 evidences that only those farmers with a low risk aversion ($\lambda < \lambda_{lim}$) will be willing to invest in soil biodiversity in a rainfed system. However, a slightly higher risk aversion but still below λ_L (i.e. $\lambda_{lim} < \lambda < \lambda_L$) would provide them a negative expected utility so that they will prefer to stop farming this crop.

Table 3:summary of decisions intervals driven by risk aversion when the drought frequency is high

	$\lambda < \lambda_L$		$\lambda > \lambda_L$	
	$\lambda < \lambda_{lim}$	$\lambda > \lambda_{lim}$	$\lambda < \lambda_0$	$\lambda > \lambda_0$
Risk aversion				
Expected utility	$R_B > R_A$ $R_B > 0$	$R_B > R_A$ $R_B < 0$	$R_A > R_B$ $R_A > 0$	$R_A > R_B$ $R_A < 0$
Production	$F > 0$	$F = 0$	$F > 0$	$F = 0$
Soil biodiversity	$I_b = I_b^*$	$I_b = 0$	$I_b = 0$	$I_b = 0$
Strategy	Rainfed farming using soil biodiversity (Option B)	Stop farming the crop (Option C)	Rainfed farming without soil biodiversity (Option A)	Stop farming the crop (Option C)

Therefore, when droughts are relatively more frequent, only those farmers with a small enough risk aversion ($\lambda < \lambda_{lim}$) will invest in soil biodiversity conservation provided that this natural insurance is not too costly compared to the output price.

For the biodiversity conservation policies to be effective, it is important to deal with farmers' heterogeneity regarding their risk aversion and how they are distributed relatively to those thresholds. For example, if a group of farmers has a level of risk aversion just below λ_L, the policy maker may incite them to conserve soil biodiversity with less cost than if the risk aversion level are more spread. The frequency of drought also affects the way risk averse farmers behave. When the drought frequency is relatively low, more risk averse farmers are likely to invest in soil biodiversity conservation practices but as the frequency of drought becomes high those farmers will not invest in soil biodiversity conservation since the water transfer function of soil biodiversity is less likely to operate and provide them with a natural insurance. Another insurance for farmers is the use of irrigation under strategy D.

3.3 The farmer invests in soil biodiversity conservation and irrigation

We recall that it has been assumed that if the period is humid (with rainfall π_h) the plant water needs are satisfied without irrigation and that if the period is dry (defining a drought) irrigation is banned by the regulator. So irrigation never takes place in period 2.

The use of nonlinear production function enables to simplify the problem defined by equations (6)-(8) as:

$$\max_{I_b,w_1} \varphi_h p F_{max}[1 - \lambda \varphi_l p F_{max}] - c_b I_b - c_w w_1 \tag{17}$$

Under the constraint of water demand

$$(1+\alpha)\pi_l + \frac{(\gamma - \alpha)\pi_l + w_1}{1+\beta} \geq \bar{X} \tag{$I1$}$$

and the total water quantity that cannot be higher than the soil saturation capacity:

$$\alpha\pi_l + w_1 \leq L' I_b^{\mu'} \tag{$I2$}$$

3.3.1 An analytical solution

We solve this problem analytically and then we provide a graphical interpretation of the solutions in section 3.3.2. To solve this problem analytically, let us first suppose that the water quantity equals the soil saturation capacity, i.e. constraint $(I2)$ is saturated **so** $w_1' = L' I_b^{\mu'} - \alpha\pi_l$.

The problem (17) becomes the following:

$$\max_{I_b} \varphi_h p F_{max}[1-\lambda\varphi_l p F_{max}] - c_b I_b - c_w\left(L' I_b^{\mu'} - \alpha\pi_l\right)$$

under the constraint of plant water demand:

$$(1+\alpha)\pi_l + \frac{(\gamma-\alpha)\pi_l + w_1}{1+\beta} \geq \bar{X} \tag{$I1$}$$

This maximization problem is equivalent to the following minimization problem:

$$\min_{I_b} c_b I_b + c_w\left(L' I_b^{\mu'} - \alpha\pi_l\right) \tag{18}$$

under the following constraints:

$$L' I_b^{\mu'} \geq \bar{X} - (1+\gamma-\alpha)\pi_l + \frac{k[\bar{X}-(1+\alpha)\pi_l]}{L} I_b^{-\mu} \tag{$I1$}$$

The minimization problem (18) would have no solution without the water demand constraint $(I1)$. Therefore, the solution to this problem is the solution of the following equation:

$$L' I_b^{\mu'} = \bar{X} - (1+\gamma-\alpha)\pi_l + \frac{k[\bar{X}-(1+\alpha)\pi_l]}{L} I_b^{-\mu} \tag{19}$$

This equation has a unique solution. Let us denote I_b' its solution.

We can conclude that when the soil saturation constraint $(I2)$ is saturated then the water demand constraint $(I1)$ will necessarily be saturated. And the solution of the problem is the couple ($I_b', L' I_b'^{\mu'} - \alpha\pi_l$).

Now let us suppose that the water demand constraint $(I1)$ is saturated and the soil saturation constraint $(I2)$ not necessarily saturated.

Then the following water demand equation must be satisfied:

$$(1+\alpha)\pi_l + \frac{(\gamma-\alpha)\pi_l + w_1}{1+\beta} = \bar{X}$$

Implying $w_1 = (1+\beta)[\bar{X}-(1+\alpha)\pi_l] - (\gamma-\alpha)\pi_l$.

The original maximization problem becomes the following minimization problem:

$$\min_{I_b} c_b I_b + c_w\left(\left(1+\frac{k}{L}I_b^{-\mu}\right)(\bar{X}-(1+\alpha)\pi_l)-(\gamma-\alpha)\pi_l\right) \tag{20}$$

under the following constraint:

$$\alpha\pi_l + w_1 \leq L' I_b^{\mu'} \tag{I2}$$

The level of biodiversity that solves the problem (20) is the following, provided $(I2)$ is respected:

$$I_b'' = \left[\frac{\mu k c_w}{L c_b}(\bar{X}-(1+\alpha)\pi_l)\right]^{\frac{1}{1+\mu}} \tag{21}$$

For the saturation constraint $(I2)$ to be respected we must have:

$$L' I_b^{\mu'} \geq \bar{X}-(1+\gamma-\alpha)\pi_l + \frac{k[\bar{X}-(1+\alpha)\pi_l]}{L} I_b^{-\mu} \tag{22}$$

- If $I_b'' \geq I_b'$, then the condition (22) is satisfied and I_b'' is the solution of the problem.
- If $I_b'' \leq I_b'$, then the condition (22) is not satisfied. In that case I_b' from equation (19) is the solution to the problem (20). Note that I_b' depends only on parameters such as rainfall and different coefficient and not on the relative costs while the value of I_b'' in equation (21) depends on the relative costs of soil biodiversity and irrigation water.

3.3.2 Farmers sensitivity to the relative cost of inputs

We provide a graphical interpretation of the problem. On the same graph, we plot the frontier lines of the water demand and soil saturation constraints and represent the domain where the constraints are satisfied. The equations representing the frontier curves of the constraints are:

$$w_1 = \left(1+\frac{k}{L I_b^{\mu}}\right)(\bar{X}-(1+\alpha)\pi_l)-(\gamma-\alpha)\pi_l \tag{23}$$

$$w_1 = L' I_b^{\mu'} - \alpha\pi_l \tag{24}$$

Equation (23) refers to the minimum productive level of water and equation (24) refers to the saturation point.

These two equations are plotted in a graph relating soil biodiversity with water in order to delimit the area where the two constraints are satisfied (shaded area in Figure 2 and Figure 3). The area below the water demand curve does not respect the minimum water quantity necessary for agricultural production. Above the curve the water demand constraint is respected. In the area above the soil saturation curve the water quantity is above the saturation point. Below the curve, the water quantity is below the saturation point and the soil saturation constraint is satisfied. The shaded area is then the area where both constraints are satisfied. Possible solutions to the farmer's problem are in that area.

The cost objective function can be formulated by the following expression: $z = c_b I_b + c_w w_1$ where z corresponds to a given level of the objective function. The resolution of the farmer's problem consists in finding the minimum value of z that satisfies the minimum productive water constraint and the soil saturation constraint for a given value of input unit costs, c_b and c_w. The isocost line $w_1 = -\frac{c_b}{c_w} I_b + z$ represent different combinations of water and biodiversity that will result in the same level of cost. We consider different values of the relative cost $\frac{c_b}{c_w}$. Figure 2 represents the case of a high relative cost of soil biodiversity. Figure 3 the case of a low relative cost of soil biodiversity.

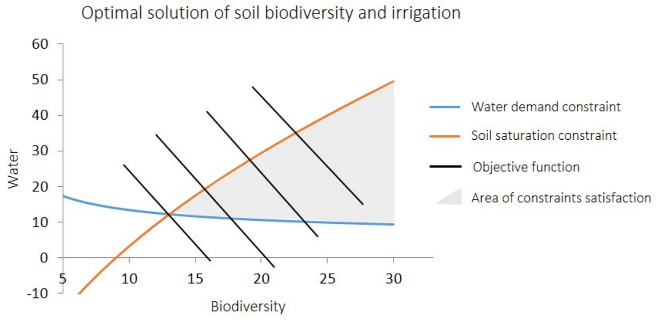


Figure 2:Representation of the objective function for high values of the relative cost c_b/c_w

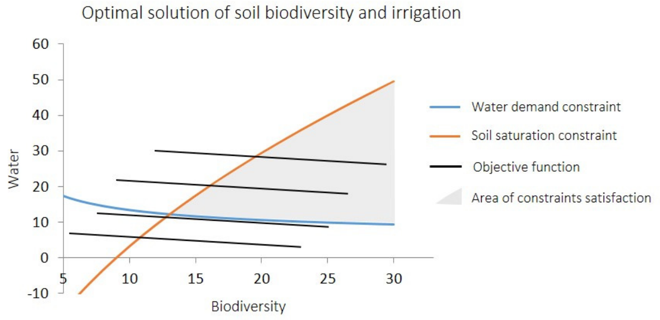


Figure 3: Representation of the objective function for low values of the relative cost c_b/c_w

In Figure 2 the solution is the lowest value of the costs function that satisfies the different constraints. For a given relative costs ratio, the slope of the objective function is unchanged. The higher values of the objective function are represented by lines in the upper-right side of the graph. The lowest value of the objective function that satisfies the constraints is the one whose representative line crosses the intersection of the constraints curves. Therefore, the corresponding soil biodiversity and water values are the optimal choice. This is the couple $(I'_b, w'_1) = \left(I'_b, L' I_b'^{\mu'} - \alpha\pi_l\right)$ previously determined in the analytical resolution (section 3.3.1).

In Figure 3 the ratio c_b/c_w is low meaning that soil biodiversity costs much less than water. The slope of the objective function is thus much less steep. The isocost line for which the cost function is minimal does not pass through the intersection point of the two constraints curves anymore. The objective function is now tangent to the curve representing the water needs constraint. This means that a solution is obtained when the derivative of the objective function line (its slope) equals the derivative of the water need equation. This condition is written as:

$$-\frac{c_b}{c_w} = -\frac{\mu k}{L I_b^{\mu+1}}(\bar{X} - (1+\alpha)\pi_l) \tag{25}$$

which implies that

$$I_b = \left[\frac{\mu k c_w}{L c_b}(\bar{X} - (1+\alpha)\pi_l)\right]^{\frac{1}{1+\mu}} = I_b'' \tag{26}$$

This value of soil biodiversity is the same as I_b'' in equation (21) found when we analytically solved the problem. The associated water level is:

$$w_1'' = \left(1 + \frac{k}{L\left[\frac{\mu k c_w}{L c_b}(\bar{X} - (1+\alpha)\pi_l)\right]^{\frac{\mu}{1+\mu}}}\right)(\bar{X} - (1+\alpha)\pi_l) - (\gamma - \alpha)\pi_l \tag{27}$$

Therefore, depending on the size of the relative cost c_b/c_w, the optimal levels of soil biodiversity and irrigation are independent of their costs. It then exists a relative cost c^* which defines the sensitivity of the optimal levels of inputs to their costs. If the relative cost is beyond this costs threshold, (i.e. $\frac{c_b}{c_w} > c^*$) then I_b' and w_1' are the solutions to the problem and these levels of inputs are independent of the costs of soil biodiversity and irrigation. The farmer is insensitive to relative costs. On the contrary if the relative cost is below the costs threshold (i.e., $\frac{c_b}{c_w} < c^*$), then the quantities I_b'' and w_1'' are the solutions to the problem and are sensitive to the cost of inputs (Figure 4).

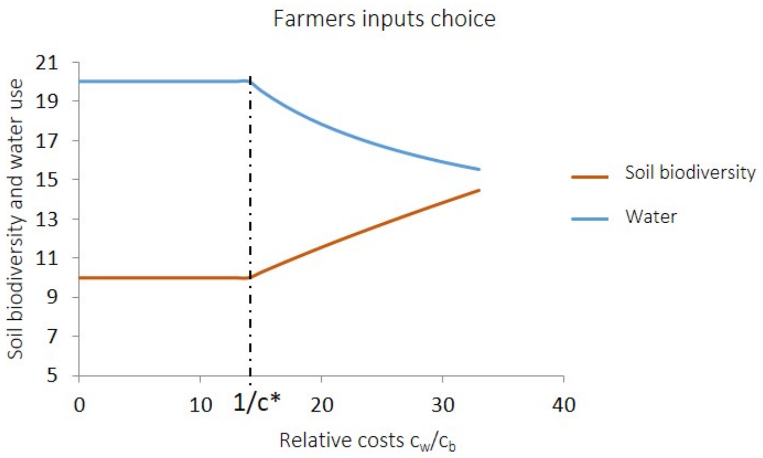


Figure 4:Representation of farmers optimal input use strategy as a function of $1/c^* = c_w/c_b$

A change of the costs of inputs will affect the profit and the expected utility. However, only a substantial change (depending on the quantity $\frac{c_b}{c_w} - c^*$) would affect the choice of inputs and make the farmer to revise his/her optimal level of inputs. An substantial increase of water cost or a decrease of soil biodiversity costs will move the farmer context from a situation where $\frac{c_b}{c_w} > c^*$ to another where $\frac{c_b}{c_w} < c^*$ in which he/she would become sensitive to costs as illustrated Figure 4.

An increase of soil biodiversity cost (similarly a decrease of water cost) should lead the farmer to substitute water to biodiversity. We would therefore expect the farmer to use more water and less soil biodiversity in order to have the same amount of productive water X over the two periods. But when $\frac{c_b}{c_w} > c^*$, if the farmer uses more water and less soil biodiversity, the soil saturation constraint will no more be satisfied[6]. Therefore, the additional water will be wasted. And, if soil biodiversity conservation cost is too high the farmer will eventually stop using soil biodiversity. But in that case, the farmer will have to stop irrigation too in order to satisfy the soil saturation constraint[7]. The farmer will therefore

---

[6] When $\frac{c_b}{c_w} > c^* \Leftrightarrow \mathrm{c_w}/\mathrm{c_b} < 1/c^*$, the optimal solution is the couple $(I_b', L' I_b'^{\mu'} - \alpha\pi_l)$ for which the soil saturation (I2) and the water demand (I1) constraints are saturated.

[7] We recall that an assumption is that irrigation is banned during a drought period. So to use non rainfall water in period 2 the farmer must use water transferred by soil biodiversity.

count only on natural rainfall but it has been shown (section 3.2) that if the farmer is strongly risk averse he/she might just decide to stop farming the crop.

On the other hand, a decrease of the cost of soil biodiversity (an increase in the cost of water) should lead to more soil biodiversity use. But if $\frac{c_b}{c_w} > c^*$, the cost of soil biodiversity is still too high for the farmer to prefer substituting soil biodiversity to water (thus reduce irrigation) and relying on water transfer by soil biodiversity. If he/she did so, the water demand constraint would not be satisfied, the minimum productive water level could not be reached. In other words, when $\frac{c_b}{c_w} > c^*$ the farming system is more supported by the contribution of irrigation to the water demand constraint than by the water transfer service provided by soil biodiversity[8]. Therefore, the soil biodiversity level remains unchanged until the threshold c* is crossed. Table 4 summarizes the optimal inputs levels in the different contexts of costs ratio.

Table 4: Summary of the results

	If $\boldsymbol{\frac{c_b}{c_w} \geq c^*}$	**If $\boldsymbol{\frac{c_b}{c_w} < c^*}$**
Optimal soil biodiversity	I_b' as the solution to the equation $L'I_b^{\mu'}$ $= \bar{X} - (1 + \gamma - \alpha)\pi_l$ $+ \frac{k[X_l - (1 + \alpha)\pi_l]}{L} I_b^{-\mu}$	$I_b^{''} = \left[\frac{\mu k c_w}{L c_b}(\bar{X} - (1 + \alpha)\pi_l)\right]^{\frac{1}{1+\mu}}$
Optimal irrigation	$w_1' = L'I_b'^{\mu'} - \alpha\pi_l$	$w_1^{''} = \left(1 + \frac{k}{L\left[\frac{\mu k c_w}{L c_b}(\bar{X} - (1 + \alpha)\pi_l)\right]^{\frac{\mu}{1+\mu}}}\right)(\bar{X}$ $- (1 + \alpha)\pi_l) - (\gamma - \alpha)\pi_l$
Relation with costs	I_b' and w_1' do not depend on c_w and c_b. Changes in costs don't affect optimal decisions.	$I_b^{''}$ and $w_1^{''}$ depend on c_w and c_b. Changes in costs affect the optimal decisions
Threshold of relative costs	For higher values of c_w/c_b, the optimal value of soil biodiversity switches from I_b' to $I_b^{''}$ and the optimal irrigation from w_1' to $w_1^{''}$	

---

[8] Because in this context of cost structure the irrigation level is the one that saturates the soil saturation constraint.

Change in relative costs	A decreased water cost (resp an increase in biodiversity cost) will not affect biodiversity level (resp water) level. Conversely, an increase in water cost (a decrease in biodiversity cost) will not affect the farmers' decisions up to a certain threshold value c*.	Any change of relative costs will affect the farmer input decisions.

4 Discussion

These results present important policy implications for water management. If we consider that in arid regions, the cost of irrigation is likely to be much higher than the cost of soil biodiversity conservation practices, because either the water tariff accounts for the resource opportunity costs or the high infrastructure cost, then farmers are likely to be sensitive to price changes and to respond to a tax on water by a different level of inputs. But in non-arid regions, where the cost of irrigation is likely to be low, a price-based instrument will have a limited effect unless the tax is high enough to reverse the cost ratio condition. The allocation of water quotas is more likely to produce changes in inputs uses.

Quotas might be preferable to price-based instruments because farmers' optimal choice of inputs is likely to be insensitive to price changes if the change in the cost ratio is weak but farming profitability will be burdened (Molle, 2009). As a consequence, when the access to irrigation water is restricted with the implementation of quotas and prohibition of irrigation during drought periods, the farmer will use more biodiversity in order to compensate for the decrease in his/her available allocated water. In fact, although the water quantity that the farmer may use is reduced, using more biodiversity would enable a better conservation of soil moisture and therefore contribute in keeping the production level constant.

The environmental conditions such as the nature of the soil, the plant water needs, the climate and the water transfer capacity of soil biodiversity determine the threshold that makes farmers sensitive to input price regulation. Coupling of soil biodiversity conservation policies to water resource management policies could help to manage both water and soil biodiversity. Indeed, soil biodiversity is difficult to observe and to regulate (Lemanceau et al., 2015) while water quotas are simpler to quantify and to monitor. Thence, the quota is a driver of soil biodiversity investment by the farmer as it determines the threshold that provides the rationale to use the water regulation function of soil biodiversity so as to choose the optimal solution closed to $\left(I_b^{''}, w_1^{''}\right)$.

Defining a quota would also require to account for the heterogeneity in biological conditions; the agronomic and soil characteristics of the farming system are key drivers of the water need constraint and of the threshold of inputs costs, as per equation (25). A sensitivity of the model to different soil type, k, minimum water needs $\bar{X}$ and water demand α is proposed in Appendix 6.2. Sandy soils would require higher level of soil biodiversity and irrigation than clay soils to satisfy the minimum water need constraint. Crops with high productive water requirements will also need more soil biodiversity and water. Growing plants that uptake more water in the first period than in the second would require less soil biodiversity and water as the water transfer is less essential for plant growth.

5 Conclusion

A link between soil biodiversity, water regulation ecosystem service and an economic rational of farming is established and illustrated with a bioeconomic model. This model represents the decision making made by a utility maximizing farmer with risk aversion that faces nonlinear production with threshold; the production is effective only beyond a certain level where water needs by the plant is satisfied. Such farmer has to decide between farming in a rainfed or irrigated system with or without the implementation of soil biodiversity conservation practices. The optimal decision depends on how large is the farmers' risk aversion compared to different thresholds where one strategy dominates the other or produces a negative expected utility, and on how large is the ratio of the cost of soil biodiversity to the cost of water. Two key elements can be highlighted from this study.

First, when both irrigation and soil biodiversity conservation practices are used for production, the farmer sensitivity to price-based policies depends upon the relative cost ratio of these two inputs. For farmers to change their optimal level of inputs after an input price-based policy, such as a tax or a subsidy, the cost of soil biodiversity conservation might be much smaller than the cost of irrigation, i.e. the ratio of cost of soil biodiversity to the cost of water below a certain threshold, otherwise farmers do not react to marginal changes of input prices but suffer a reduced profitability.

Second, the decisions of farmers depend on exogenous factors such as the intrinsic nature of the soil, the function of soil biodiversity, the climatic conditions and the agricultural system. These factors also determine the thresholds that we discussed. Finding these thresholds is a matter of further empirical investigation.

6 Appendices

6.1 Proofs and calculations

6.1.1 The farmer decides to grow crops without using biodiversity

Proof of $R_A = \varphi_h^2 pF_{max}\left[1 - \lambda\left(1 - \varphi_h^2\right)pF_{max}\right]$**:**

We recall the assumption $(1+\alpha)\pi_l < \bar{X} < 2\alpha\pi_l$ and the condition $X_1 + X_2 \geq \bar{X}$ for a non-zero production.

State of Nature	X_1	X_2	$X_1 + X_2$	Is $X_1 + X_2 \geq \bar{X}$ satisfied ?	$F(X_1, X_2)$
SN1	π_l	π_l	$2\pi_l$	No, since $X_1 + X_2 = 2\pi_l < (1+\alpha)\pi_l < \bar{X}$	0
SN2	π_l	$\alpha\pi_l$	$(1+\alpha)\pi_l$	No, since $X_1 + X_2 = (1+\alpha)\pi_l < \bar{X}$	0
SN3	$\alpha\pi_l$	$\alpha\pi_l$	$2\alpha\pi_l$	Yes, since $X_1 + X_2 = 2\alpha\pi_l > (1+\alpha)\pi_l$	F_{max}
SN4	$\alpha\pi_l$	π_l	$(1+\alpha)\pi_l$	No, since $X_1 + X_2 = (1+\alpha)\pi_l < \bar{X}$	0

We have:

$$E\big(pF(X_1, X_2)\big) = p\left(\varphi_l^2 F_1(\pi_l, \pi_l) + \varphi_l\varphi_h F_2(\pi_l, \alpha\pi_l) + \varphi_l\varphi_h F_4(\alpha\pi_l, h\pi_l) + \varphi_h^2 F_3(\alpha\pi_l, \alpha\pi_l)\right)$$

And $F_1(\pi_l, \pi_l) = F_4(\alpha\pi_l, \pi_l) = F_2(\pi_l, \alpha\pi_l) = 0 \; and \; F_3(\alpha\pi_l, \alpha\pi_l) = F_{max}$.

Therefore

$$E\big(pF(X_1, X_2)\big) = p\left(\varphi_h^2 F_{max}\right) = p\varphi_h^2 F_{max}$$

Similarly, we have:

$$Var\big(pF(X_1, X_2)\big) = p^2\left(\left(\varphi_h^2 F_{max}^2\right) - \left(\varphi_h^2 F_{max}\right)^2\right) = p^2\varphi_h^2\left(1 - \varphi_h^2\right)F_{max}^2$$

6.1.2 The farmer uses a certain level of soil biodiversity and no irrigation.

Proof of $R_B = \varphi_h pF_{max}[1 - \lambda\varphi_l pF_{max}] - c_b {I_b}^*$**:**

We recall the assumption $(1+\alpha)\pi_l < \bar{X} < 2\alpha\pi_l, \; 1 \leq \alpha \leq \gamma$ and the condition $X_1 + X_2 \geq \bar{X}$.

State of Nature	X_1	X_2	X_1+X_2	Is $X_1+X_2 \geq \bar{X}$ satisfied ?	$F(X_1,X_2)$
SN1	π_l	π_l	$2\pi_l$	No, since $X_1+X_2 = 2\pi_l<(1+\alpha)\pi_l<\bar{X}$	0
SN2	π_l	$\alpha\pi_l$	$(1+\alpha)\pi_l$	No, since $X_1+X_2 = (1+\alpha)\pi_l<\bar{X}$	0
SN3	$\alpha\pi_l$	$\alpha\pi_l$	$2\alpha\pi_l$	Yes, since $X_1+X_2 = 2\alpha\pi_l>(1+\alpha)\pi_l$ since $1\leq\alpha$	F_{max}
SN4	$\alpha\pi_l$	$h\pi_l$	$(\alpha+h)\pi_l$	Yes, since $X_1+X_2 = (\alpha+h)\pi_l > (1+\alpha)\pi_l$ since $h\geq 1$ as $\alpha\leq\gamma$	F_{max}

We have:

$$E\big(pF(X_1,X_2)\big) = p\left(\varphi_l^2F_1(\pi_l,\pi_l) + \varphi_l\varphi_hF_2(\pi_l,\alpha\pi_l) + \varphi_l\varphi_hF_4(\alpha\pi_l,h\pi_l) + \varphi_h^2F_3(\alpha\pi_l,\alpha\pi_l)\right),$$

and $F_1(\pi_l,\pi_l) = F_2(\pi_l,\alpha\pi_l) = 0$ and $F_4(\alpha\pi_l,h\pi_l) = F_3(\alpha\pi_l,\alpha\pi_l) = F_{max}$.

Therefore

$$E\big(pF(X_1,X_2)\big) = p\big(\varphi_l\varphi_hF_{max} + \varphi_h^2F_{max}\big) = p\varphi_h(\varphi_l+\varphi_h)F_{max} = p\varphi_hF_{max}$$

Similarly, we have:

$$Var\big(pF(X_1,X_2)\big) = p^2\left(\big(\varphi_l\varphi_hF_{max}^2 + \varphi_h^2F_{max}^2\big) - \big(\varphi_h^2F_{max}\big)^2\right) = p^2\varphi_l\varphi_hF_{max}^2$$

Proof of the condition for $R_B - R_A > 0$

$$\begin{aligned} R_B - R_A &= \varphi_hpF_{max}[1-\lambda\varphi_lpF_{max}] - c_bI_b{}^* - \varphi_h^2pF_{max}\big[1-\lambda\big(1-\varphi_h^2\big)pF_{max}\big] \\ &= \varphi_hpF_{max}\left[1-\varphi_h-\lambda pF_{max}\left(\varphi_l-\varphi_h\big(1-\varphi_h^2\big)\right)\right] - c_bI_b{}^* \\ &= \varphi_l\varphi_hpF_{max}\left[1+\lambda pF_{max}\big(\varphi_l^2-3\varphi_l+1\big)\right] - c_bI_b{}^* \end{aligned}$$

$$R_B - R_A > 0 \iff \lambda > \frac{1}{pF_{max}(\varphi_l^2-3\varphi_l+1)}\left[\frac{c_bI_b{}^*}{\varphi_l\varphi_hpF_{max}} - 1\right]$$

6.2 Sensitivity of the arbitrage with respect to biological conditions

The sensitivity of the model with respect to the properties of the soil, the nature of the agricultural system, the potential evapotranspiration is analyzed.

- **Change in the intrinsic nature of the soil, k.**

The parameter k measures the infiltration capacity of the soil made of a given type of material. For example, while we would expect a sandy soil to possess high values of k, clay soils are more likely to have low values of the infiltration capacity. Figure A 1 shows how the water constraint curves and the optimal solution change when k increases from a clay soil k_1 to a sandy soil k_3. High values of k shifts up the minimum water constraint. For higher values of k, the soil will be less capable of retaining water. Therefore, to obtain the production level F_{max}, the farmer needs to put more water and conserve more soil biodiversity in order to help the soil to retain water until the second period. Farmers who use soils with a higher value of k (e.g. sandy soils) will require at the same time more biodiversity and more water at the optimal point all other things being equal particularly the relative costs.

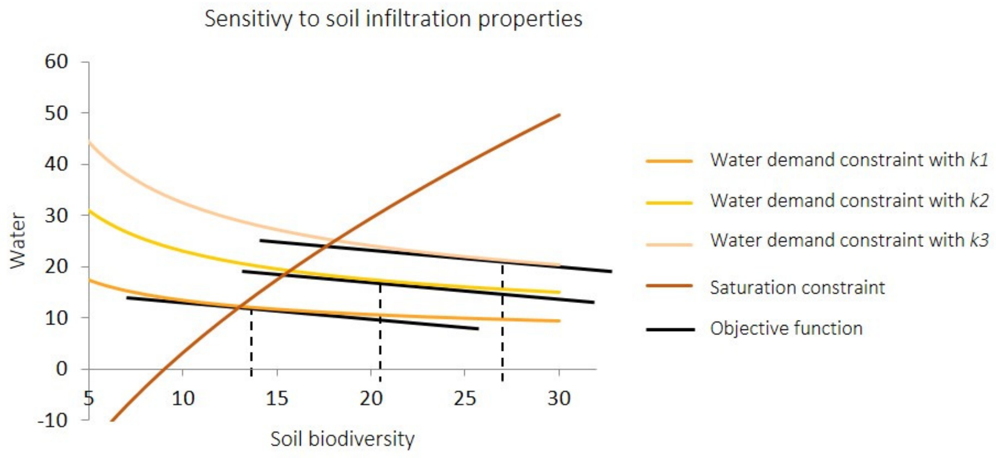


Figure A 1 : Sensitivity to soil properties, k

- **Changes in the plant water needs, $\overline{X}$.**

Figure A 2 represents the constraints for different level of the minimum productive water level $\overline{X}$. It shows that high values of $\overline{X}$ shifts the minimum water need constraint up. Farmers who grow crops with high productive water requirements will need at the same time more soil biodiversity and more water at the optimal point all other things being equal, particularly the relative costs. The higher need of both inputs for plants with higher water requirements insures the transfer of a certain amount of that water from the first to the second period.

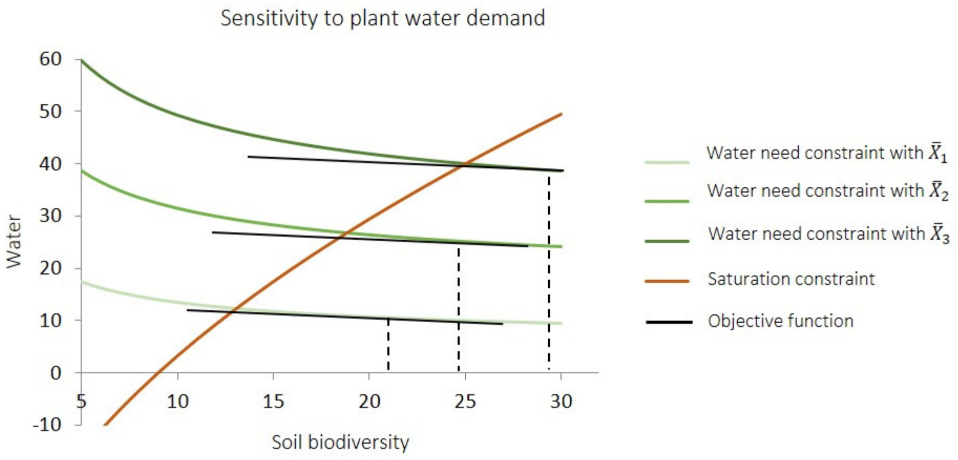


Figure A 2: Sensitivity to water plant demands, $\bar{X}$

- **Changes in Potential Evapotranspiration α**

Figure A 3 illustrates how the optimal inputs level and the water need constraint change as the potential evapotranspiration, α changes. Higher values of α move the water constraints curves downward which leads to lower values of irrigation and soil biodiversity. An increased value of α means that the plants will uptake more water at the first period when the availability of water is high. It is better to uptake water at the first period than at the second period because some of the water in the soil is lost between the first and the second period. Therefore, a plant that uptakes more water at the first period (higher α), all other things being kept equal, will need less irrigation hence less soil biodiversity because the minimum limit $\bar{X}$ will be more easily reached.

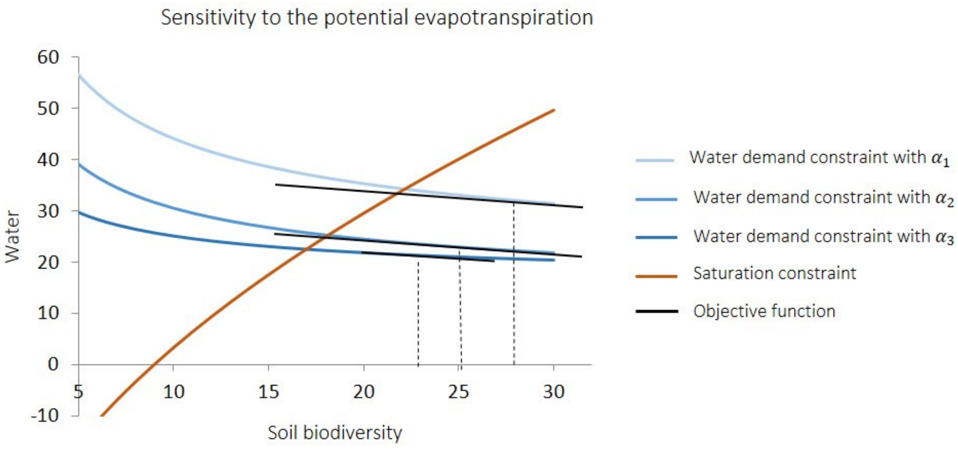


Figure A 3: Sensitivity to the potential evapotranspiration α

7 References

Allison, F.E., 1973. Soil Organic Matter and its Role in Crop Production, Elsevier. ed.

Altieri, M.A., 1999. The ecological role of biodiversity in agroecosystems. Agriculture, Ecosystems & Environment 74, 19–31. https://doi.org/10.1016/S0167-8809(99)00028-6

Bardgett, R.D., Anderson, J.M., Behan-Pelletier, V., Brussaard, L., Coleman, D.C., Ettema, C., Moldenke, A., Schimel, J.P., Wall, D.H., 2001. The Influence of Soil Biodiversity on Hydrological Pathways and the Transfer of Materials between Terrestrial and Aquatic Ecosystems. Ecosystems 4, 421–429. https://doi.org/10.1007/s10021-001-0020-5

Bardgett, R.D., Putten, W.H. van der, 2014. Belowground biodiversity and ecosystem functioning. Nature 515, 505–511. https://doi.org/10.1038/nature13855

Bastardie, F., Ruy, S., Cluzeau, D., 2005. Assessment of earthworm contribution to soil hydrology: a laboratory method to measure water diffusion through burrow walls. Biol Fertil Soils 41, 124–128. https://doi.org/10.1007/s00374-004-0817-4

Baumgärtner, S., 2007. The Insurance Value of Biodiversity in the Provision of Ecosystem Services. Natural Resource Modeling 20, 87–127. https://doi.org/10.1111/j.1939-7445.2007.tb00202.x

Chavas, J.-P., 2008. On the economics of agricultural production*. Australian Journal of Agricultural and Resource Economics 52, 365–380. https://doi.org/10.1111/j.1467-8489.2008.00442.x

Coble, K.H., Barnett, B.J., 2013. Why Do We Subsidize Crop Insurance? Am J Agric Econ 95, 498–504. https://doi.org/10.1093/ajae/aas093

Crowther, T.W., Hoogen, J. van den, Wan, J., Mayes, M.A., Keiser, A.D., Mo, L., Averill, C., Maynard, D.S., 2019. The global soil community and its influence on biogeochemistry. Science 365. https://doi.org/10.1126/science.aav0550

De Deyn, G.B., Van der Putten, W.H., 2005. Linking aboveground and belowground diversity. Trends in Ecology & Evolution 20, 625–633. https://doi.org/10.1016/j.tree.2005.08.009

Di Falco, S., Chavas, J.-P., 2009. On Crop Biodiversity, Risk Exposure, and Food Security in the Highlands of Ethiopia. Am J Agric Econ 91, 599–611. https://doi.org/10.1111/j.1467-8276.2009.01265.x

Eisenhauer, N., 2012. Aboveground–belowground interactions as a source of complementarity effects in biodiversity experiments. Plant Soil 351, 1–22. https://doi.org/10.1007/s11104-011-1027-0

Esteve, P., Varela-Ortega, C., Blanco-Gutiérrez, I., Downing, T.E., 2015. A hydro-economic model for the assessment of climate change impacts and adaptation in irrigated agriculture. Ecological Economics 120, 49–58. https://doi.org/10.1016/j.ecolecon.2015.09.017

Foudi, S., Erdlenbruch, K., 2012. The role of irrigation in farmers' risk management strategies in France. Eur Rev Agric Econ 39, 439–457. https://doi.org/10.1093/erae/jbr024

Groom, B., Koundouri, P., Nauges, C., Thomas, A., 2008. The story of the moment: risk averse cypriot farmers respond to drought management. Applied Economics 40, 315–326. https://doi.org/10.1080/00036840600592916

Hatfield, J.L., Sauer, T.J., Prueger, J.H., 2001. Managing Soils to Achieve Greater Water Use Efficiency: A Review. AGRONOMY JOURNAL 93, 11.

Kirkham, M.B., 2005. Principles of soil and plant water relations, Elsevier. ed.

Koundouri, P., Nauges, C., Tzouvelekas, V., 2006. Technology Adoption under Production Uncertainty: Theory and Application to Irrigation Technology. Am J Agric Econ 88, 657–670. https://doi.org/10.1111/j.1467-8276.2006.00886.x

Lemanceau, P., Maron, P.-A., Mazurier, S., Mougel, C., Pivato, B., Plassart, P., Ranjard, L., Revellin, C., Tardy, V., Wipf, D., 2015. Understanding and managing soil biodiversity: a major challenge in agroecology. Agron. Sustain. Dev. 35, 67–81. https://doi.org/10.1007/s13593-014-0247-0

Letey, J., Dinar, A., Woodring, C., Oster, J.D., 1990. An economic analysis of irrigation systems. Irrig Sci 11, 37–43. https://doi.org/10.1007/BF00189993

Levy, H., Markowitz, H.M., 1979. Approximating Expected Utility by a Function of Mean and Variance. The American Economic Review 69, 308–317.

Lubbers, I.M., Brussaard, L., Otten, W., Groenigen, J.W.V., 2011. Earthworm-induced N mineralization in fertilized grassland increases both N2O emission and crop-N uptake. European Journal of Soil Science 62, 152–161. https://doi.org/10.1111/j.1365-2389.2010.01313.x

Mantovani, E.C., Villalobos, F.J., Organ, F., Fereres, E., 1995. Modelling the effects of sprinkler irrigation uniformity on crop yield. Agricultural Water Management 27, 243–257. https://doi.org/10.1016/0378-3774(95)01159-G

Markowitz, H., 2014. Mean–variance approximations to expected utility. European Journal of Operational Research, 60 years following Harry Markowitz's contribution to portfolio theory and operations research 234, 346–355. https://doi.org/10.1016/j.ejor.2012.08.023

Milly, P.C.D., Betancourt, J., Falkenmark, M., Hirsch, R.M., Kundzewicz, Z.W., Lettenmaier, D.P., Stouffer, R.J., 2008. Stationarity Is Dead: Whither Water Management? Science 319, 573–574. https://doi.org/10.1126/science.1151915

Molle, F., 2009. Water scarcity, prices and quotas: a review of evidence on irrigation volumetric pricing. Irrig Drainage Syst 23, 43–58. https://doi.org/10.1007/s10795-009-9065-y

Nielsen, U.N., Ayres, E., Wall, D.H., Bardgett, R.D., 2011. Soil biodiversity and carbon cycling: a review and synthesis of studies examining diversity-function relationships. Eur. J. Soil Sci. 62, 105–116. https://doi.org/10.1111/j.1365-2389.2010.01314.x

Nielsen, U.N., Wall, D.H., Six, J., 2015. Soil Biodiversity and the Environment. Annual Review of Environment and Resources 40, 63–90. https://doi.org/10.1146/annurev-environ-102014-021257

Pascual, U., Termansen, M., Hedlund, K., Brussaard, L., Faber, J.H., Foudi, S., Lemanceau, P., Jørgensen, S.L., 2015. On the value of soil biodiversity and ecosystem services. Ecosystem Services 15, 11–18. https://doi.org/10.1016/j.ecoser.2015.06.002

Roscoe, K.H., 1968. Soils and model tests. Journal of Strain Analysis 3, 57–64. https://doi.org/10.1243/03093247V031057

Sidibé, Y., Foudi, S., Pascual, U., Termansen, M., 2018. Adaptation to Climate Change in Rainfed Agriculture in the Global South: Soil Biodiversity as Natural Insurance. Ecological Economics 146, 588–596. https://doi.org/10.1016/j.ecolecon.2017.12.017

Spurgeon, D.J., Keith, A.M., Schmidt, O., Lammertsma, D.R., Faber, J.H., 2013. Land-use and land-management change: relationships with earthworm and fungi communities and soil structural properties. BMC Ecology 13, 46. https://doi.org/10.1186/1472-6785-13-46

Turbé, A., De Toni, A., Lavelle, P., Benito, P., Lavelle, Patrick, Ruiz, N., Van der Putten, W.H., Mudgal, S., 2010. Soil biodiversity: functions, threats and tools for policy makers: final report. Bio Intelligence Service, IRD, and NIOO, Report for European Commission (DG Environment), Paris.

Wagg, C., Bender, S.F., Widmer, F., Heijden, M.G.A. van der, 2014. Soil biodiversity and soil community composition determine ecosystem multifunctionality. PNAS 111, 5266–5270. https://doi.org/10.1073/pnas.1320054111

Wall, D.H., Nielsen, U.N., Six, J., 2015. Soil biodiversity and human health. Nature 528, 69–76. https://doi.org/10.1038/nature15744

Wezel, A., Casagrande, M., Celette, F., Vian, J.-F., Ferrer, A., Peigné, J., 2014. Agroecological practices for sustainable agriculture. A review. Agron. Sustain. Dev. 34, 1–20. https://doi.org/10.1007/s13593-013-0180-7