

# Grid regulation with heterogeneous prosumers

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The objective of this paper is to study the integration to the energy grid of heterogeneous prosumers. We consider a setting where both the metering technology and tariffs have an influence on the decision of consumers to invest in photovoltaic panels to become prosumers and on the cost recovery of the grid operations. We show that a net metering system, where prosumers import and export energy from/to the grid at the same price, leads to too much PV investments compared to the first best. In addition, with this technology, we find that consumers are attracted into prosumption independently from their self-consumption profile, i.e. the extent with which their energy consumption correlates with their production. When a net purchasing system is in place, where prosumers import and export energy from/to the grid at different prices, consumers with a relatively high self-consumption profile are more likely to invest in PV installations, which is desirable from the energy system viewpoint as they exchange less with the grid. Even when the regulator is not informed about the self-consumption rate of prosumers, net purchasing is able to reach the first best outcome.

**Keywords:** Decentralized production unit, grid regulation, solar panel, grid tariff.

**JEL Codes:** D13, L51, L94, Q42

## 1 Introduction

The electricity sector is involved in major transformations worldwide. Both the liberalization of the upstream and downstream markets and the emergence of renewable energy sources as photovoltaics and windmills are the drivers of the development for future ways to consume and produce electricity. The level of cost-competitiveness reached by PV devices allows users to move to self-consumption decisions concerning electricity and eventually to peer-to-peer exchanges of the self-produced energy. This new trend

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called "prosumption" is facilitated by technical developments such as battery systems smart meters and advanced business models promotes self-consumption via the technical design of the electricity systems. The success of these developments depends, however, on the regulatory and administrative framework in terms of energy policy and regulation, grid financing, taxation and legal relationships amongst the involved entities and it requires innovative solutions coupled with suitable business and management models to achieve a sustainable system integration.

Households deciding to invest in photovoltaic panels tend to remain connected to the energy grid in order to keep having access to electricity, even when the weather conditions are not favorable for them to produce electricity. According to Mckenna et al. (2018), they consume on average 45% of the electricity they produce, the rest being exported to the grid and is used in order to satisfy the demand of other consumers. Behind this figure there are wide disparities. As production is in large part exogenous due to the level of solar radiations, differences in self-consumption rates are for the most due to differences in consumption profiles. For example, a household working from home during daytime will self-consume much more than a household away from home during daytime.

From the grid perspective, self-consumption is important as it allows to minimize grid-related costs due to grid reinforcement investments or energy losses related to congestion. Unfortunately, for the moment, smart meters have not been generalized yet and precise information concerning the self-consumption rate of households is limited. When prosumers are integrated to the grid by the mean of a net-metering system, this information cannot be inferred from the meter, even ex-post, as only the consumption net of the production exported to the grid is measured. As a consequence, fixing tariffs to encourage behaviors that are beneficial from the perspective of the energy system is rendered more complicated.

The goal of this paper is to study how the heterogeneous profile of prosumers with respect to self-consumption interacts with the way tariffs are chosen. We study separately the two ways that are commonly used in order to integrate decentralized production units into the energy system: the net metering and the net purchasing systems. The first is a meter with only one line to compute the energy exchanges that will turn backwards as soon as energy not consumed at its place of production is exported to the grid. The second has two lines that measure separately imports and exports of electricity. We analyze how different tariff schemes will encourage consumers to self-select in order to become prosumers. We also study whether efficient tariffs can be implemented when the information concerning self-consumption is unknown to the grid operator and whether prosumers will truthfully reveal their type when choosing among different tariffs.

Our main results are the following. First, we show that consumers more inclined to self-consume tend to be more likely to invest in PV installations to become prosumers when a net purchasing system is in place. In the case of a net metering system, this investment decision is independent from self-consumption. The reason is that, in the former case, the energy exported to the grid is less valued than the energy imported

from the grid, which is here measured on a separate line of the meter. Hence, we show that the net meter not only encourages too much investments into decentralized production units but also that the wrong types of consumers, from the wider point of view of the energy system, are attracted in this kind of investment. Our second key result is that, even if the distribution system operator has no information regarding the rate of self-consumption of prosumers, efficient tariffs can be set when net purchasing is implemented and tariffs differ with respect to this self-consumption rate.

This paper is closely related to the literature analyzing the interactions between decentralized production units and a network operator, especially via the financing of the former (Burger et al. (2018)). The key idea is that how tariffication is implemented in order to cover the (mostly fixed) network costs will send signals to consumers whether or not to invest in photovoltaics. As in Brown and Sappington (2017) or Schittekatte et al. (2018), we assume that both the choice of the meter and the network tariff design can impact this decision. More precisely, we build upon a previous model by Gautier et al (2018) where consumers also differ with respect to how much their consumption profile is correlated with their production profile in the case they have a PV installation. Thanks to this assumption, we are able to discuss the types of the households who decide to self-select into prosumption by investing in PV and compare it with a centrally planned decision. Under a net metering system, we show that the right types do not make a good investment decision from the point of view of the system. Hence, our work is also related to the one of Wagner (2019) who develops a spatial economics model to show that, in a decentralized setting, the locational choice of renewable production units will not always be optimal.

This paper also contributes to a second dimension of the literature focusing on public utility regulations, concerned by the tariff structure. According to the classical result of Coase (1946), two-part tariffs with volumetric charges set to marginal costs and a fixed charge to recoup the fixed costs of the monopoly lead to an efficient outcome. Due to the presence of prosumers with differing self-consumption profiles, one of our key result is that two-part tariffs need to diverge from Coasian tariffs. To be optimal, i.e. attract the efficient amount and type of prosumers, the fixed part of the tariff need to cover more than the fixed costs of the network operator. In addition, this fixed part should depend on whether or not the household has a PV installation.

The paper is organized as follows. Section 2 provides a presentation of the model and the first best outcome is presented in section 3. Section 4 highlights the two key policies to be chosen by the monopoly distribution system operator. Then, while section 5 presents the net metering case, section 6 presents the net purchasing one. Section 7 concludes.

## 2 Model

This paper builds upon the model developed in Gautier et al. (2018) of a separated and regulated Distribution System Operator (DSO) that operates the network activities as a monopoly. There are two other key players in the energy system: centralized electricity producers and consumers who can also become producers by investing in a Decentralized Production Unit (DPU). By choosing the way DPU are integrated into the energy system and the prices they pay for the exchanges with the grid, different amount and profiles of consumers will decide to invest.<sup>1</sup>

**Consumers** There is a population of consumers of size 1. All consumers have a fixed consumption  $q$ , giving them a surplus  $S$ . Consumers can install a DPU of capacity  $\tilde{k}$  that produces an energy flow  $k = \beta\tilde{k}$  where  $\beta$  is the load factor of a consumer. The cost of an installation of size  $\tilde{k}$  is equal to  $\tilde{z}\tilde{k}$ . For a prosumer the cost of producing  $k$  is equal to  $zk$  with  $z = \tilde{z}\beta$ . We suppose that all prosumers produce the same quantity  $k$  but that they have a different installation cost  $z$  and we consider an independent and logconcave distribution  $f(z)$  on a closed interval,  $[\underline{z}, \bar{z}]$ . Without loss of generality, we normalize  $\underline{z} = 0$  and  $\bar{z} = 1$ .

Compared to Gautier et al. (2018), we introduce a second source of heterogeneity among consumers, the degree of self-consumption  $\varphi$ . Prosumers self-produces  $k$  but only a fraction  $\varphi k$  of the local production is self-consumed, the remaining fraction  $(1 - \varphi)k$  being exported to the grid.  $\varphi$  measures the degree of correlation between the timing of consumption and the timing of production of the decentralized production units. On the one hand, when perfectly correlated, everything that is produced by the PV installation is consumed at the place of production. On the other hand, when  $\varphi = 0$ , all the decentralized production is exported to the grid. This variable tends to be very heterogeneous according to the literature on self-consumption (see McLaren et al. (2015), Luthander et al. (2015) and Lang et al. (2016)). Focusing on different contexts and using different methods, McKenna et al. (2018) and Gautier et al. (2019) explain this difference in ratios by the day-time electricity usage at home. For example, a household working from home will have a much higher self-consumption rate than someone working away from home. Likewise, there is more self-consumption in office building than in residential houses. In the model, we represent self-consumption by the synchronization factor  $\varphi \in [0, 1]$ , independently distributed by  $g(\varphi)$  between these two extreme self-consumption profile.

We assume that the two parameters  $z$  and  $\varphi$  are independently distributed one from the other.<sup>2</sup> Each consumer is identified by two variables  $(z, \varphi)$  and the consumers are

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<sup>1</sup>Recently, Dato et al. (2020) have also studied how regulations impact decentralized investments in renewable energy production units. The authors are mainly concerned by the possibility given to prosumers to sell the energy fed into the grid and the installation of smart meters. In addition, they are not concerned by the financing of the DSO while it is the key focus of this paper.

<sup>2</sup>This independence assumption allows a simplified presentation of the results but these hold if we

distributed on a square of size 1, with on the vertical axis the cost of installing a DPU  $z$  and on the horizontal axis the correlation factor  $\varphi$  (see Figure 1). We will represent the status of a consumer  $(z, \varphi)$  by a binary variable  $x(z, \varphi) \in \{0, 1\}$ . If  $x(z, \varphi) = 1$  the agent is a prosumer and produces  $k$  and if  $x(z, \varphi) = 0$  it is not.

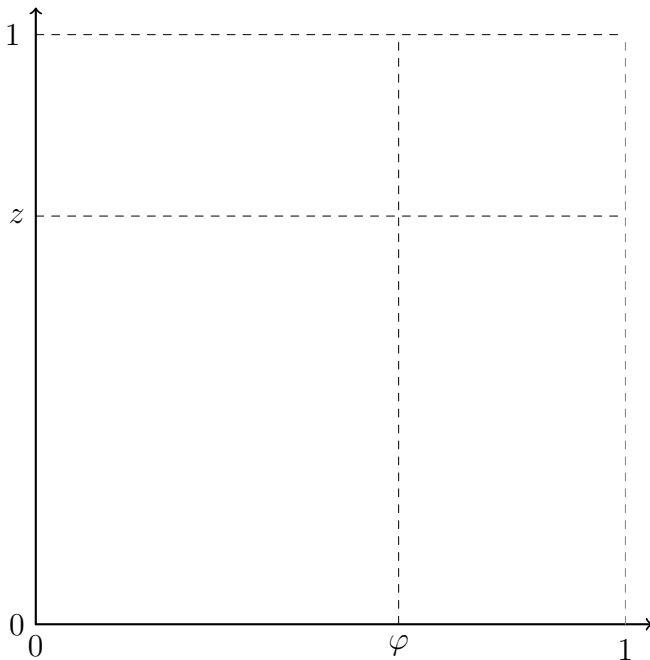


Figure 1: Distribution of preferences

**Costs of generation** The total consumption is equal to  $q$  as the population size is equal to 1. This consumption is covered by the production of centralized and decentralized production units. CPU produces energy at unit cost  $c$  and we assume that  $c$  lies in between  $\underline{z} = 0$  and  $\bar{z} = 1$ . Without prosumers, the total generation cost is  $C_g = cq$ .

The total production of the DPU is equal to:  $k \int_0^1 \int_0^1 x(z, \varphi) f(z) g(\varphi) d\varphi dz$  and the corresponding cost of decentralized production is

$$C_g^{DPU} = k \int_0^1 \int_0^1 x(z, \varphi) z d\varphi dz$$

We suppose that centralized and decentralized production are perfect substitutes. The 

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impose a more general bivariate law  $f(z, \varphi)$ .

total generation cost is equal to:

$$\begin{aligned} C_g &= \int_0^1 \int_0^1 \{(1 - x(z, \varphi)) cq + x(z, \varphi) [zk + c(q - k)]\} f(z) g(\varphi) d\varphi dz \\ &= cq + k \int_0^1 \int_0^1 x(z, \varphi) (z - c) f(z) g(\varphi) d\varphi dz \end{aligned}$$

Notice that the generation costs are independent of the prosumers' synchronization factor.

**Grid costs** There are two types of costs for the grids: fixed costs per users and variables costs linked to the MWh of energy distributed. We denote by  $K_c$  the cost of connecting one user to the grid and, as all users are connected, the sum of these costs is equal to  $K_c$ . For a prosumer, there is an additional cost of connecting his DPU to the grid (change of meter, connection costs, etc.) and this cost is denoted by  $K_l$ .

Let us denote by  $\theta_i$  the variable cost per MWh distributed associated with centralized ( $i = c$ ) and local exchanges ( $i = l$ ). Local exchanges refer to power exchanges between DPU and other consumers and centralized exchanges refer to power exchanges between CPU and consumers/prosumers. To simplify the analysis, we will suppose that the variable costs per MWh are identical for centralized and local energy exchanges:  $\theta_l = \theta_c = \theta$ .

For each prosumer a fraction  $\varphi$  of the production of a DPU is self-consumed, the remaining fraction being exported and consumed elsewhere, so the total local distribution volume ( $V_l$ ) is then:

$$V_l = \left( \int_0^1 \int_0^1 x(z, \varphi) (1 - \varphi) f(z) g(\varphi) d\varphi dz \right) k \quad (1)$$

The total volume of centralized distribution ( $V_c$ ) is equal to the CPU production:

$$V_c = q - \left( \int_0^1 \int_0^1 x(z, \varphi) f(z) g(\varphi) d\varphi dz \right) k \quad (2)$$

Denoting the fixed cost associated with active prosumers

$$\bar{K}_l = \left( \int_0^1 \int_0^1 x(z, \varphi) f(z) g(\varphi) d\varphi dz \right) K_l$$

then the total cost of the DSO is equal to:

$$\begin{aligned} C_d &= \theta (V_c + V_l) + \bar{K}_l + K_c \\ &= \theta q + \int_0^1 \int_0^1 x(z, \varphi) (K_l - \varphi k \theta) f(z) g(\varphi) d\varphi dz + K_c \end{aligned} \quad (3)$$

We also denote  $c_d = C_d - K_c$ .

### 3 First Best

The total cost of producing and distributing electricity for the system is given by the sum of the cost of generation  $C_g$  and the cost of network distribution,  $C_d$  given above. The total cost is:

$$\begin{aligned} C &= C_g + C_d \\ &= (c + \theta)q + \int_0^1 \int_0^1 x(z, \varphi) [(z - c)k - \varphi\theta k + K_l] f(z) g(\varphi) d\varphi dz + K_c \quad (4) \end{aligned}$$

The benevolent social planner has to determine whether or not an agent characterized by a profile  $(z, \varphi)$  should invest to become a prosumer or should remain a normal consumer. Given that consumption is fixed, the first best minimizes the total costs of the energy system. This turns out to minimize  $C$  with respect  $x(z, \varphi) \in \{0, 1\}$ . A pointwise optimization shows the following.

**Proposition 1** *At the first best,  $x^*(z, \varphi) = 1$  if  $z \leq z^*(\varphi) = c + \varphi\theta - \frac{K_l}{k}$  and  $x^*(z, \varphi) = 0$  otherwise.*

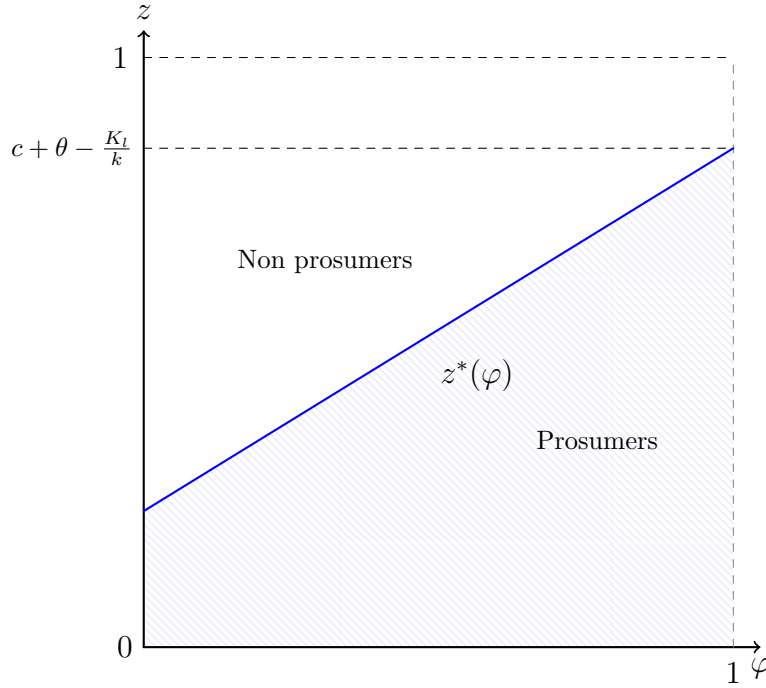


Figure 2: First best prosumption (if  $c - \frac{K_l}{k} > 0$ )

The expression for the first best is similar to Brown and Sappington (2017) and Gautier et al. (2018). The idea is that decentralized production units should be valued

at the marginal cost of the centralized generation unit net of the additional network costs created by the DPU. A DPU generates an extra connection costs but saves on grid costs as the self-consumption reduces the power exchanges with the grid.

The first best identifies a frontier  $z^*(\varphi)$ . Interestingly, the frontier is increasing in  $\varphi$  meaning that consumers with a higher installation cost  $z$  can become prosumers only if they autoconsume more. Self-consumption reduces the exchanges with the grid and the grid's cost. Hence, a higher installation cost can be compensated by a higher degree of self-consumption. This important feature is often neglected in the policy designed at encouraging the deployment of DPU. Figure 2 represents the first best frontier  $z^*(\varphi)$ .

At the first best, the optimal mass of prosumers is then  $M^* = \int_0^1 \int_0^{z^*(\varphi)} f(z) g(\varphi) dz d\varphi = \int_0^1 F(z^*(\varphi)) g(\varphi) d\varphi$ .

In the next subsections, we will look at the amount and the profile of consumers who become prosumers when this decision is decentralized and depending on the regulations in place.

## 4 Metering technologies and grid regulation

In the next sections, we look at the decentralized outcome when consumers and not the planner decide whether or not to invest in a DPU. We consider an energy system where production and distribution are separated. Production is a competitive activity and the energy price  $p$  is equal to the marginal cost of centralized production units:  $p = c$ . Distribution is a monopolistic activity and an energy regulator fixes the tariff of the DSO and organizes the power exchanges between the prosumers and the grid.

### 4.1 Metering technologies

Prosumers are making two types of exchange with the grid. They inject their production surplus when the DPU produces more than the instantaneous consumption and they withdraw power when their production is insufficient to cover their instantaneous consumption. A prosumer with a synchronization factor  $\varphi$ , exports  $(1 - \varphi)k$  to the grid and imports  $q - \varphi k$  from the grid.

In jurisdictions equipped with smart meters, the meter records the two power flows separately. Without smart meters, there are either two mechanical meters that record imports and exports or a single meter that runs backwards when the energy is exported to the grid (net-metering). In the latter case, the single meter records the net imports  $q - k$ .

The synchronization factor  $\varphi$  and the total consumption  $q$  are not directly observed on the meter readings. To compute them, the prosumer needs an additional meter



to record its production  $k$ .<sup>3</sup> Then  $q$  and  $\varphi$  can be recovered *ex-post* from the meter readings. The export meter records  $(1 - \varphi)k$  from which the synchronization can be computed. And the total consumption  $q$  can be computed from the import meter that records  $q - \varphi k$ .

## 4.2 Net metering and net purchasing

When a consumer purchases electricity from the grid, he pays the commodity price  $p$  and a network fee  $r^m > 0$  that we call the import fee. When a consumer injects electricity to the grid, he is paid the commodity price  $p$  and he pays in addition an injection fee  $r^x$  that we call the export fee. This export fee can be positive, negative (in which case exports are subsidized by the DSO), or nil.

Net metering is a commonly used pricing system system in which the imports and the exports have the same value i.e. the price paid for imports is equal to the price received for exports. As discussed in Moura and Brito (2019), this metering technology is among others the one in place in most U.S. states, in Mexico, Brazil, Finland, India and Belgium. The implementation of net metering requires to set the network fees such that  $r_m = -r_x$ . Under net purchasing, the two power flows, imports and exports, will have a different value for the prosumers which requires, given that the commodity price is the same for exports and imports,  $r_m \neq r_x$ . Notice that when the prosumer is equipped with a single mechanical meter to records its exchanges with the grid, net metering is the only possible option.

## 4.3 Tariff structure

The tariff is regulated and it must be set at a level that allows the DSO to recover its total cost  $C_d$  with its receipts.

We consider different tariff structures:

- A Coasian two-part tariff where each consumer pays a fixed fee equal to the grid's fixed cost  $\rho = K_c$  and variables fees are designed to cover the variable costs  $c_d$ .
- A Coasian tariff with a prosumer fee where the prosumers pay an additional fixed fee to cover the prosumers' specific fixed cost. In this tariff structure, traditional consumers pay  $\rho = K_c$  and prosumers pay  $\rho = K_c + K_p$ .<sup>4</sup>

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<sup>3</sup>In the case of solar PV, the installation of a meter on the inverter can be used to measure the production of the DPU.

<sup>4</sup>Cambini and Soroush (2019) discuss and compare different tariff structure under net metering. They focus in particular on the recovering of the fixed connection costs linked to DPU. The Coasian tariff corresponds to what they call the 'shallow connection cost' where fixed connection costs are recovered by the variable part of the tariff. The Coasian tariff with a prosumer's fixed fee corresponds to the 'deep connection cost'. They show that deep connection cost charging achieves better result than shallow connection costs.

- A non Coasian two-part tariff where the tariff is designed to minimize the total cost of the system ( $C_d + C_g$ ).

This exercise will be done for both net metering and net purchasing. *jc Pourquoi le rejeter d'emble du moins pour net purchasing alors que c'est une consequence de la tarification ?* Finally, we suppose in line with the discussion above that the regulator does not observe  $\varphi$ , i.e. the regulator has no access to the prosumers' production meter (if any). Therefore, it cannot make the network tariff contingent on the synchronization factor and use this variable to screen among prosumers. Furthermore, we will prove that a menu of contract where prosumers select their preferred option has no value added in this context.

## 5 Net metering

### 5.1 The prosumer's investment decision

Consider the net metering case with a unique variable fee  $r = r^m = -r^x$  and a fixed fee  $\rho$  per consumer. As in Gautier et al. (2018), the net utility of consumer able to install a DPU producing  $k \leq q$  and auto-consuming at a rate  $\varphi$  is given by:

$$U(z, \varphi) = \begin{cases} S - (c + r)(q - k) - \rho - zk & \text{if } x(z, \varphi) = 1 \\ S - (c + r)q - \rho & \text{if } x(z, \varphi) = 0 \end{cases}$$

The consumer indifferent between investing and not investing in a DPU is such that

$$\tilde{z} = c + r \tag{5}$$

Hence, we have

$$\begin{aligned} \tilde{x}(z, \varphi) &= 1 \text{ if } z \leq c + r \\ \tilde{x}(z, \varphi) &= 0 \text{ if } z > c + r \end{aligned}$$

With net metering, the self-consumption level does not affect the consumer's investment decision. The reason is that a net-metering system does not differentiate the price of the two energy flows and the prosumer's bill and its utility depend only on the net imports  $q - k$ , independently of the synchronization level.

Therefore, as we rule out tariffs contingent on  $\varphi$ , we can establish that:

**Proposition 2** *It is not possible to implement the first best with net metering.*

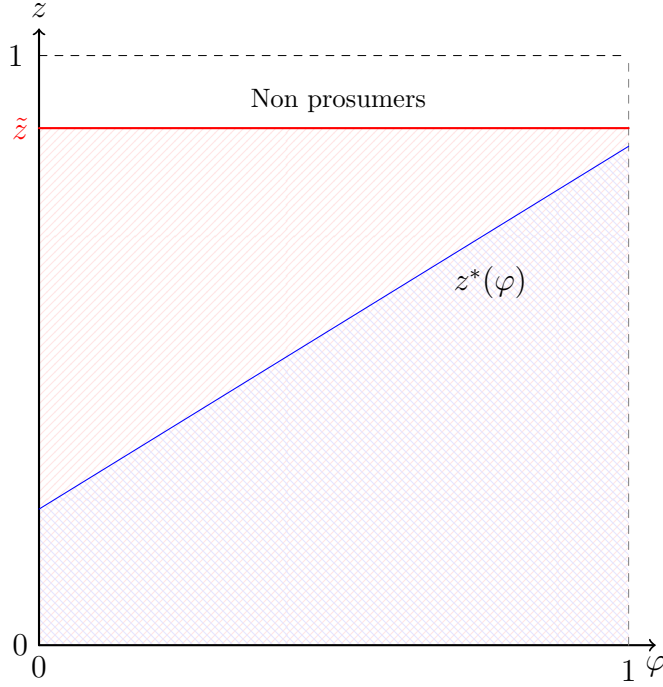


Figure 3: Prosumers under net metering (Coasian tariff)

## 5.2 Coasian tariff

With a Coasian tariff ( $\rho = K_c$ ), the recorded consumption volume is  $V_c$  and the break even distribution tariff  $\tilde{r}$  satisfies:

$$\tilde{r}(z) = \frac{c_d}{V_c}$$

Given that  $\tilde{z} = c + r$ , the proportion of prosumers in the population is  $F(\tilde{z}) = F(c + r)$ . Replacing the cost  $c_d$  and the import volume  $V_c$  by their values evaluated at  $\tilde{z}$ , the break even grid fee is given by:

$$\tilde{r} = \frac{\theta q + F(\tilde{z})(K_l - \theta \bar{\varphi} k)}{q - F(\tilde{z})k} \quad (6)$$

where  $\bar{\varphi} = \int_0^1 \varphi g(\varphi) d\varphi$  is the mean value of  $\varphi$  i.e. the average self-consumption level in the society. The resulting cut-off value  $\tilde{z} = c + \tilde{r}$  is represented in Figure 3. We observe that the cut-off value is higher than the first best  $\tilde{z} > z^*$  for all  $\varphi \in [0, 1]$  and it is independent of  $\varphi$ .

**Proposition 3** *Compared to the first best, net metering with a Coasian tariff induces a too important mass of prosumers.*

This result is similar to idea of the death spiral described Brown and Sappington (2017) and Gautier et al. (2018). The idea is that tariffs do not send the right signal to consumers when they make their investment decision. The true benefits of their installation is over-evaluated as the price at which they sell their energy to the network, because they do not consume it, is the same as the price at which they buy electricity from the network. Furthermore, as prosuming is encouraged too much, this uniform tariff rate has to increase, otherwise the DSO will not break even, and this further encourages households to become prosumers.

### 5.3 Coasian tariff with a prosumer fixed fee

To limit the number of prosumers, the regulator can rebalance the tariff structure either by imposing a specific fixed fee to the prosumers or by increasing the fixed fee (next subsection).

In the first case, the regulator imposes a fixed fee  $\rho_p = K_c + K_l$  to prosumers and  $\rho = K_c$  to consumers. With such a fee, the cut-off investment level is now:

$$\tilde{z}' = c + r - \frac{K_l}{k}$$

and the corresponding break even grid fee is:

$$\tilde{r}' = \theta \frac{q - F(\tilde{z}') k \bar{\varphi}}{q - F(\tilde{z}') k}$$

Combining the two equations, we have:

$$\tilde{z}' = z^*(\varphi) + \theta \frac{(1 - \varphi) q - F(\tilde{z}') k (\bar{\varphi} - \varphi)}{q - F(\tilde{z}') k}$$

And we can straightforwardly show that it exists

$$\tilde{\varphi} = \frac{q - F(\tilde{z}) k \bar{\varphi}}{q - F(\tilde{z}) k} > \bar{\varphi}$$

such that

$$\begin{aligned} \tilde{z}' &\geq z^*(\varphi) \text{ if } \varphi \leq \tilde{\varphi} \\ \tilde{z}' &< z^*(\varphi) \text{ if } \varphi > \tilde{\varphi} \end{aligned}$$

Under this tariff, prosumers pay for the fixed connection cost of the DPU. As a consequence, the variable fee can be reduced ( $r' < \tilde{r}$ ) and the death spiral is mitigated as. The tariff is now based upon the average variable cost evaluated for the average prosumers which exhibits a self-consumption rate of  $\bar{\varphi}$ . As a result now, prosumers which higher self-consumption rates are discouraged by this tariff structure and there are less prosumers. Figure 4 represents the consumers' investment decision.

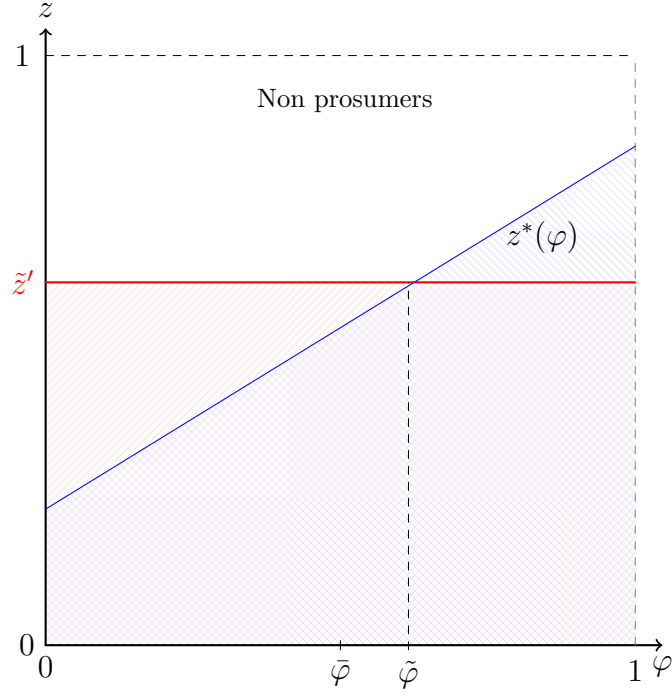


Figure 4: Prosuming with Coasian tariff and prosumer fee

## 5.4 Optimal two-part tariff

Instead of charging a specific fixed fee for DPU, the regulator can decrease the variable fee  $r$  and compensate by increasing the fixed fee  $\rho$  to limit the benefit of net metering and lower the investment in DPU while guaranteeing that the DSO realizes a non-negative profit. The optimal two-part tariff under net metering is the solution of:

$$\min_{r,\rho} C \text{ subject to } z = r + c \text{ and } \rho + r(q - F(r + c)k) = C_d$$

The solution to this problem is to set the following unit tariff

$$\tilde{r} = \bar{\varphi}\theta - \frac{K_l}{k} - H(\tilde{r} + c) < \bar{\varphi}\theta - \frac{K_l}{k}$$

where  $H(z) = \frac{F(z)}{f(z)} \geq 0$  and increasing<sup>5</sup> in  $z$ , and to cover the remaining costs of the DSO with the fixed fee

$$\tilde{\rho} = \left( (1 - \bar{\varphi})\theta + \frac{K_l}{k} \right) q + K_c + H(\tilde{r} + c) (q - F(\tilde{r} + c)k)$$

so  $\tilde{\rho} > \left( (1 - \bar{\varphi})\theta + \frac{K_l}{k} \right) q + K_c$ . And we can show easily that it exists

$$\tilde{\varphi} = \bar{\varphi} - \frac{H(\tilde{z})}{\theta} < \bar{\varphi}$$

<sup>5</sup>This is due to the logconcavity of  $f(z)$ .

such that

$$\begin{aligned} \tilde{z} &\geq z^*(\varphi) \text{ if } \varphi \leq \tilde{\varphi} \\ \tilde{z} &< z^*(\varphi) \text{ if } \varphi > \tilde{\varphi} \end{aligned}$$

Under this tariff, all consumers and prosumers pay more than the fixed connection cost of the DPU. As a consequence, the variable fee can be reduced ( $\tilde{r} < \tilde{r}$ ) and the death spiral is mitigated deeply. The tariff is now based *below* the average variable cost evaluated for the average prosumers which exhibits a self-consumption rate of  $\tilde{\varphi}$ . This is to take into account the **externality/total effect of z** ?. As for the coasian tariff with a prosumer fixed fee, prosumers which higher self-consumption rates are discouraged by this tariff structure and there are less prosumers.

## 5.5 Comparisons

None of the three tariff structure we considered manages to achieve the first best.

Finally, we can easily show that a dedicated fixed prosumer fee improves the performance of the energy system as it decreases its total cost. First, we show:

**Lemma 1**  $\tilde{r} < \tilde{r}' < \tilde{r}$  and  $\tilde{z} < \tilde{z}' < \tilde{z}$

Proof: see Appendix.

With a given net metering tariff, the total cost is:

$$C(r, z) := (c + \theta)q + \int_0^1 \int_0^z [rk - \varphi\theta k + K_l] f(y) g(\varphi) dy d\varphi + K_c$$

which increases with both  $(r, z)$ , given Lemma 1, We have  $\tilde{C} < \tilde{C}' < \tilde{C}$

\*\*\* Masses We cannot clearly compare prosumers masses for both last solutions. It depends on the particular distribution of  $z$ .

\*\*\* Finally, we should mention that a menu of contract  $(r(\varphi), \rho(\varphi))$  where the prosumers pick their preferred tariff according to their synchronization level is ineffective. Indeed, as the prosumers' utility is independent of  $\varphi$ , all prosumers will pick the same tariff and it is impossible to screen prosumers' according to their synchronization factor.

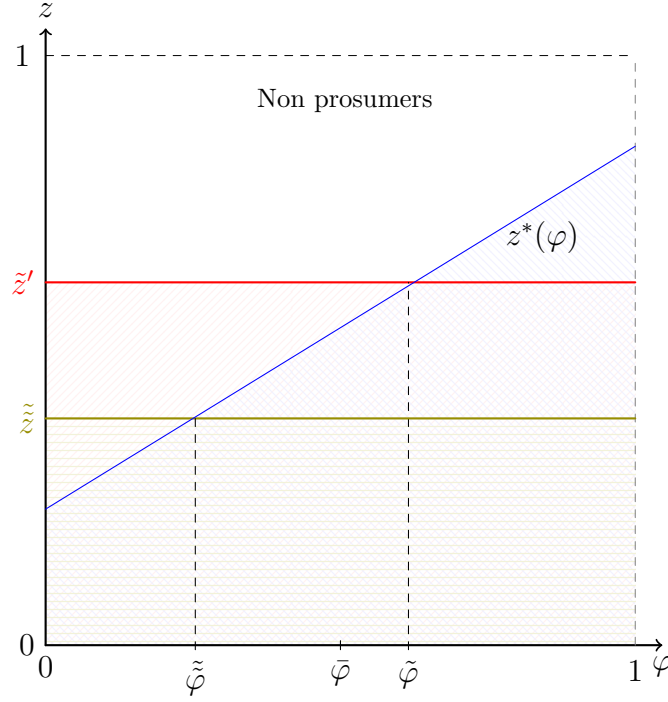


Figure 5: Comparisons

## 6 Net purchasing

### 6.1 The prosumer's investment decision

Under a net purchasing scheme, the net utility of a prosumer who can install a DPU producing  $k$  and with a self-consumption level  $\varphi$  is given by is

$$U(z) = \begin{cases} S - c(q - k) - r_m(q - \varphi k) - r_x(1 - \varphi)k - \rho - zk & \text{if } x(z, \varphi) = 1 \\ S - (c + r_m)q - \rho & \text{if } x(z, \varphi) = 0 \end{cases}$$

### 6.2 Coasian tariff

Again, let us first consider a Coasian tariffs such that  $\rho = K_c$ . The consumer who is indifferent is such that

$$\hat{z}(\varphi) = c + \varphi r_m - (1 - \varphi) r_x. \quad (7)$$

Now we see that only agents that have preferences  $(z, \varphi)$  such that  $z \leq \hat{z}(\varphi)$ , will become prosumers. It is now clear that net purchasing can only implement the first-best level of prosumers if for all all agents  $\varphi \leq 1$ .

$$\varphi(r_m + r_x) - r_x = \varphi\theta - \frac{K_l}{k}$$

that is if

$$r_m = \theta - \frac{K_l}{k} \quad \text{and} \quad r_x = \frac{K_l}{k} \quad (8)$$

However this tariff structure is not profitable for the DSO.

**Proposition 4** *Under a net metering system, Coasian tariffs cannot implement the first best.*

This result contrasts with Gautier et al. (2018) where a net purchasing system with a uniform tariff was shown to be efficiency inducing. We have that the import rate must be set below the marginal cost  $\theta$  and the export rate is used to fully charge prosumers the fixed distribution cost of a DPU installation. This makes it impossible to construct a tariff that both is fully cost reflective and induces an efficient deployment of DPU. The main issue is now that the source of heterogeneity in the society is also coming from the different degrees of correlation between consumption and production of the DPU owner.

As an efficiency inducing uniform tariff is not feasible, let us consider a second-best uniform Coasian tariff structure under a net purchasing scheme that ensures profitability for the DSO and minimizes the total cost defined in (4). In other words, we look for  $(r_m, r_x)$  that solve the problem:

$$\begin{cases} \min_{r_m, r_x} C \\ \text{s.t. } \Pi = r_m(V_c + V_l) + r_x V_l - c_d \geq 0 \\ \text{with } x(z, \varphi) = 1 \text{ if } z \leq \hat{z}(\varphi) = c + \varphi r_m - (1 - \varphi) r_x, \text{ for all } \varphi \in [0, 1] \end{cases} \quad (9)$$

We solve this problem in the appendix which allows us to state the following result, using the following notations  $\hat{M} = \int_0^1 F(\hat{z}(\varphi)) g(\varphi) d\varphi$  and  $\hat{\Phi} = \int_0^1 \varphi F(\hat{z}(\varphi)) g(\varphi) d\varphi$ .

**Proposition 5** *The second-best uniform tariff structure with net purchasing entails*

$$\hat{r}_m^* = \theta - \hat{\Gamma} \frac{K_l}{k} \quad \text{and} \quad \hat{r}_x(\hat{r}_m^*) = \frac{(\theta - \hat{r}_m^*) (q - k\hat{\Phi}) + K_l \hat{M}}{k (\hat{M} - \hat{\Phi})}$$

where  $\hat{\Gamma} < 1$ .

The second best uniform tariff allows to define the prosuming locus  $\hat{z}^*(\varphi) = c + \varphi \hat{r}_m^* - (1 - \varphi) \hat{r}_x(\hat{r}_m^*)$  which is steeper then the optimal locus.

So it may exist  $\hat{\varphi} = \varphi^*(\hat{\varphi})$  with  $\hat{z}(\varphi) \leq z^*(\varphi)$  for  $\varphi \leq \hat{\varphi}$  and conversely. Hence the second-best uniform tariff under a net purchasing scheme favors more the highly synchronized agents as they are more likely to invest in a DPU compared with the first best.



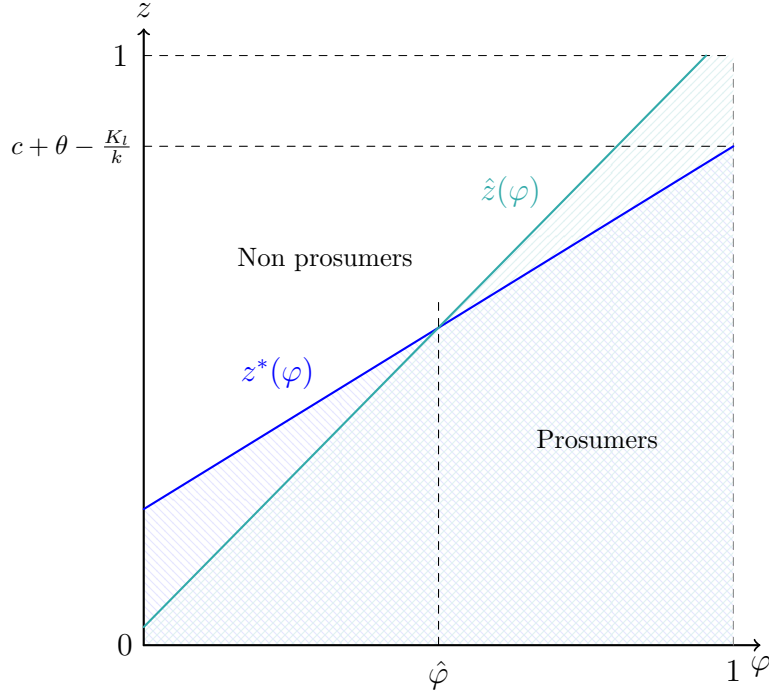


Figure 6: Second best level of prosumption under a net purchasing scheme is in green, first best is in blue

### 6.3 Optimal tariffs

With a Coasian tariff, the first best is not implemented but the regulator has different options to reach it under net purchasing. The regulator can either relax the Coasian constraint and increase the fixed fee, or it can charge different tariffs for prosumers and consumers. By doing so, it can better allocate the specific costs linked to the DPU to prosumers as proposed for instance by Cambini and Soroush (2019). As the first best can be implemented with simple tariffs, screening consumers by proposing menus of tariffs is not relevant.

First, the regulator can depart from Coasian tariffs and increase the fixed fee paid by the consumers. With  $\rho > K_c$ , it is possible to recover the grid costs and to use the grid tariff given by Equation (8) to implement the first best. The following tariff therefore implements the first best and covers the costs of the DSO:

$$r_m = \theta - \frac{K_l}{k}, r_x = \frac{K_l}{k}, \rho = K_c + \frac{K_l}{k}q \quad (10)$$

Second, the regulator can impose a discriminatory tariff and charge the energy imports of prosumers and consumers at a different rate. Let us denote by  $r_m$  and  $r_x$  the prosumers' import and export fees and by  $r_p$  the consumers' import fee. The following

Coasian tariff  $(r_m, r_x, r_c, \rho)$  implements the first best:

$$r_m = \theta + \frac{K_l}{q - k}, r_x = \frac{K_l}{q - k}, r_c = \theta, \rho = K_c \quad (11)$$

Third, the regulator can impose a dedicated fixed fee to prosumers to recover the additional costs they impose to the DSO. If we denote this fee by  $\rho_p$ , the following tariff  $(r_m, r_x, \rho_p, \rho)$  implements the first best:

$$r_m = \theta, r_x = 0, \rho_p = K_l + K_c, \rho = K_c \quad (12)$$

## 7 Conclusion

The take-up of photovoltaic investments by households can be a decentralized solution to tackle climate change by decreasing our reliance on fossil energy sources. This paper discusses various ways to integrate these investments in the energy grid, via the metering technology chosen and the way grid tariffication takes place. We find that a net metering system coupled with a two-part tariff tends to over-encourage PV investments compared to the first best. In addition to this well-known result of the literature, we show that the decision to become a prosumer is independent from how the consumption profile of prosumers is connected with their production profile, i.e. their degree of self-consumption. This is undesirable from the point of view of the energy system, as it leads to much more exchanges with the energy grid than when prosumers with a high rate of self-consumption tend to be attracted in becoming prosumers.

We further show that when a net purchasing system is in place, where for prosumers energy imports are priced differently than energy exports, the first best can be implemented even when the regulator is clueless about the self-consumption rate of prosumers. However, one condition is to depart from Coasian tariffs and, hence, have fixed charges paid to the the DSO that are higher than its fixed costs.

Potential tariff reforms are not considered in our model and merits a further analysis: time-varying tariffs or capacity tariffs. To be implemented, these tariffs require individual smart meters and, to remain non-discriminatory, most legislations require a complete roll-out of the technology. However, according to the latest figures, we are far from this goal in most European countries (ACER/CEEM (2019)) and, in some cases, at least an extra decade will be needed to reach the 80% coverage goal originally set for 2020. *julien Shall we discuss this limitation without shooting ourselves in the foot?*

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## 8 Appendix

**Proof of Proposition 1.** Straightforwardly, from the linearity of the integrand of (4) with respect to  $z$ .

**Proof of Proposition 3.** From the expression (6) in the text, we can derive  $\tilde{z} = c + \tilde{r}$  and rewrite it as:

$$\begin{aligned}\tilde{z} &= z^*(\varphi) + (1 - \varphi)\theta + (1 - \bar{\varphi})\frac{F(\tilde{z})k}{q - F(\tilde{z})k}\theta + \frac{q}{q - F(\tilde{z})k}\frac{K_l}{k} \\ &> z^*(\varphi)\end{aligned}$$

.

**Proof of Lemma 1.** (a) Comparing  $(\tilde{z}, \tilde{r})$  and  $(\tilde{z}', \tilde{r}')$  implies that

$$\begin{aligned}\tilde{r} &= \tilde{z} - c \text{ and } \tilde{r}' = \tilde{z}' - c + \frac{K_l}{k} \\ \tilde{z} &= \theta Y(\tilde{z}) + c + \frac{F(\tilde{z})}{q - F(\tilde{z})k}K_l \quad \text{and} \quad \tilde{z}' = \theta Y(\tilde{z}') + c - \frac{K_l}{k}\end{aligned}$$

where we denoting  $Y(z) = \frac{q - F(z)\bar{\varphi}k}{q - F(z)k}$  an strictly increasing positive function of  $z$  with  $Y(0) = 1$ , then for all  $z \in [0, 1]$

$$\theta Y(z) + c + \frac{F(z)}{q - F(z)k}K_l > \theta Y(z) + c - \frac{K_l}{k},$$

and since  $\theta Y(z) + c + \frac{F(z)}{q - F(z)k}K_l$  is also a strictly increasing positive function of  $z$ . This clearly shows that

$$\tilde{z}' < \tilde{z}$$

and

$$\tilde{r}' < \tilde{r}$$

(b) Comparing  $(\tilde{\tilde{z}}, \tilde{\tilde{r}})$  and  $(\tilde{z}', \tilde{r}')$  implies that

$$\begin{aligned}\tilde{r}' - \tilde{\tilde{r}} &= \theta \frac{q - F(\tilde{z}')k\bar{\varphi}}{q - F(\tilde{z}')k} - \left( \bar{\varphi}\theta - \frac{K_l}{k} - H(\tilde{\tilde{r}} + c) \right) \\ &= \theta \frac{(1 - \bar{\varphi})q}{q - F(\tilde{z}')k} + \frac{K_l}{k} + H(\tilde{\tilde{r}} + c) > 0\end{aligned}$$

so

$$\tilde{\tilde{r}} < \tilde{r}' < \tilde{r}$$

For the prosuming level

$$\tilde{z}' - \tilde{\tilde{z}} = \tilde{r}' - \frac{K_l}{k} - \tilde{\tilde{r}} = \theta \left( \frac{q(1 - \bar{\varphi})}{q - F(\tilde{z}')k} \right) + H(\tilde{\tilde{r}} + c) > 0$$

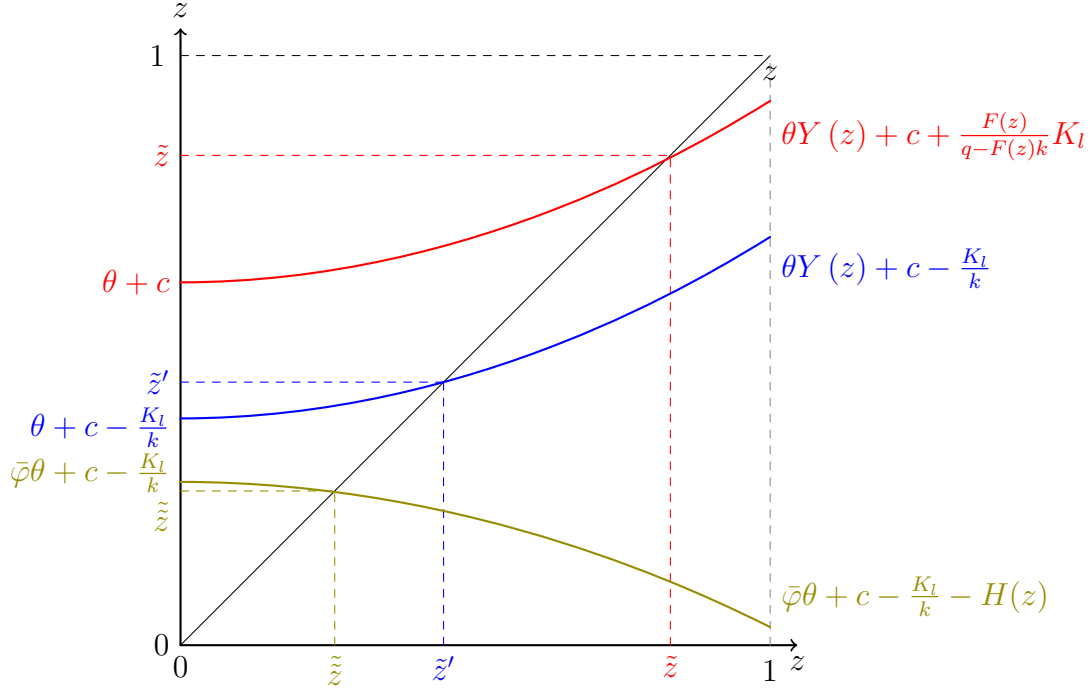


Figure 7: Comparisons of  $\tilde{z}$ ,  $\tilde{z}'$  and  $\tilde{\tilde{z}}$ .

so

$$\tilde{\tilde{z}} < \tilde{z}' < \tilde{z}$$

This can be represented in the following figure.

(c) Mass of prosumers (in progress) for a given fixed  $z$  (i.e. independant of  $\varphi$ ),  $M(z) = F(z)$  so

$$M(\tilde{\tilde{z}}) < M(\tilde{z}') < M(\tilde{z})$$

Compared to the first best mass  $M^* = \int_0^1 F(z^*(\varphi)) g(\varphi) d\varphi$ , we first know that  $M(\tilde{z}) > M^*$ . Second, it exists  $\varphi_* \in ]0, 1[$ :  $M^* = F(z^*(\varphi_*))$  so that if

$$\begin{aligned} \varphi_* > \tilde{\varphi} &\text{ then } M^* > M(\tilde{z}') > M(\tilde{\tilde{z}}) \\ \tilde{\varphi} &\geq \varphi_* > \tilde{\varphi} &\text{ then } M(\tilde{z}') \geq M^* > M(\tilde{\tilde{z}}) \\ \tilde{\varphi} &\geq \varphi_* &\text{ then } M(\tilde{z}') \geq M^* > M(\tilde{\tilde{z}}) \end{aligned}$$

This depends on the shape of the distribution  $F(z)$  for  $z$ . *je On peut aller plus loin en declinant la convexit ou concavit de  $F$  mais est-ce bien raisonnable ?*

**Proof of Proposition 4.** With the tariff structure proposed in (8) and  $\hat{\rho} = K_c$ , the DSO breakeven constraint writes

$$\Pi = r_m (\hat{V}_c + \hat{V}_l) + r_x \hat{V}_l - c_d = \left( \theta - \frac{K_l}{k} \right) \hat{V}_c + \theta \hat{V}_l - c_d$$

where from (1) and (2), the distribution volumes are:

$$\begin{aligned}\hat{V}_l &= k \int_0^1 (1 - \varphi) F(\hat{z}(\varphi)) g(\varphi) d\varphi \\ \hat{V}_c &= q - \left( \int_0^1 F(\hat{z}(\varphi)) g(\varphi) d\varphi \right) k\end{aligned}$$

and

$$\hat{c}_d = \theta q - k \int_0^1 (z^*(\varphi) - c) F(\hat{z}(\varphi)) g(\varphi) d\varphi$$

At the first-best we have  $\hat{z}(\varphi) = z^*(\varphi)$ , then rearranging the terms, the DSO account writes

$$\hat{\Pi} = -\frac{K_l}{k} q < 0$$

so the DSO cannot break even and the first-best cannot be implemented.

**Proof of Proposition 5.** Again  $\hat{\rho} = K_c$ . With net purchasing then the total cost and the DSO profit are function of  $(r_m, r_x)$  so we write:

$$\begin{aligned}\hat{C}(r_m, r_x) &= (c + \theta) q + \int_0^1 \int_0^{\hat{z}(\varphi)} [(z - c)k - \varphi k \theta + K_l] f(z) g(\varphi) d\varphi dz \\ \hat{\Pi}(r_m, r_x) &= (r_m - \theta) q - \left( \int_0^1 [\hat{z}(\varphi) - z^*(\varphi)] F(\hat{z}(\varphi)) g(\varphi) d\varphi \right) k\end{aligned}$$

Indeed at this optimum, it cannot be the case that the DSO breakeven constraint is slack, as we have seen in Proposition 4 that the cost-minimizing tariff scheme is not feasible for the DSO. So the solution will necessarily implies that  $\hat{\Pi}(r_m, r_x) = 0$ . The tariff structure that guarantees that the DSO breaks even is here (implicitly)

$$\hat{r}_x(r_m) = \frac{(\theta - r_m) (q - k\hat{\Phi}) + K_l \hat{M}}{k (\hat{M} - \hat{\Phi})} \quad (13)$$

where  $\hat{M}$  is the mass of prosumers with such a tariff structure:

$$\hat{M} = \int_0^1 F(\hat{z}(\varphi)) g(\varphi) d\varphi$$

and

$$\hat{\Phi} = \int_0^1 \varphi F(\hat{z}(\varphi)) g(\varphi) d\varphi$$

$\hat{\Phi}$  is the mean value for the degree of correlation for active prosumers only, where  $\hat{\Phi} \leq \hat{M}$ .

Differentiating (13) yields

$$\hat{r}'_x(r_m) = \frac{-\frac{(q-k\hat{\Phi})}{k} + (r_m - \theta) \hat{A} - \hat{r}_x(r_m) \hat{B} + \frac{K_l}{k} \hat{E}}{\hat{M} - \hat{\Phi} + (r_m - \theta) \hat{B} - \hat{r}_x(r_m) \hat{D} + \frac{K_l}{k} \hat{C}} \quad (14)$$

with positive constants defined as

$$\begin{aligned} \hat{A} &= \int_0^1 \varphi^2 f(\hat{z}(\varphi)) g(\varphi) d\varphi > 0; \quad \hat{B} = \int_0^1 \varphi(1-\varphi) f(\hat{z}(\varphi)) g(\varphi) d\varphi > 0 \\ \hat{C} &= \int_0^1 (1-\varphi) f(\hat{z}(\varphi)) g(\varphi) d\varphi > 0, \quad \hat{D} = \hat{C} - \hat{B} = \int_0^1 (1-\varphi)^2 f(\hat{z}(\varphi)) g(\varphi) d\varphi > 0; \\ \hat{E} &= \hat{A} + \hat{B} = \int_0^1 \varphi f(\hat{z}(\varphi)) g(\varphi) d\varphi > 0; \quad \hat{F} = \int_0^1 f(\hat{z}(\varphi)) g(\varphi) d\varphi > 0 \end{aligned}$$

with

$$\hat{C} > \hat{B}$$

Note that

$$\frac{\partial \hat{M}}{\partial r_m} = \hat{E} - \hat{r}'_x(r_m) \hat{C} \quad \text{and} \quad \frac{\partial \hat{\Phi}}{\partial r_m} = \hat{A} - \hat{r}'_x(r_m) \hat{B}$$

Now let  $\hat{C}(r_m) = \hat{C}(r_m, \hat{r}_x(r_m))$ , the FOC writes

$$\hat{C}'(r_m) = 0 \Leftrightarrow k \int_0^1 (\hat{z}(\varphi) - z^*(\varphi)) (\varphi - (1-\varphi) \hat{r}'_x(r_m)) f(\hat{z}(\varphi)) g(\varphi) d\varphi = 0$$

As from Proposition 4, it does not exist  $r_m$  such that  $\hat{z}(\varphi) = z^*(\varphi)$  when the  $\hat{\Pi}(r_m, r_x) = 0$  then  $\hat{C}'(r_m) = 0$  iff

$$\int_0^1 (\hat{z}(\varphi) - z^*(\varphi)) \varphi f(\hat{z}(\varphi)) g(\varphi) d\varphi = \hat{r}'_x(r_m) \int_0^1 (\hat{z}(\varphi) - z^*(\varphi)) (1-\varphi) f(\hat{z}(\varphi)) g(\varphi) d\varphi$$

which can also be rewritten as a differential equation in  $\hat{r}'_x(r_m)$  :

$$\hat{r}'_x(r_m) = \frac{(r_m - \theta) \hat{A} - \hat{r}_x(r_m) \hat{B} + \frac{K_l}{k} \hat{E}}{(r_m - \theta) \hat{B} - \hat{r}_x(r_m) \hat{D} + \frac{K_l}{k} \hat{C}} \quad (15)$$

Now equating (14) and (15) and substituting (13) leads to with

$$\hat{r}_m^* = \theta - \hat{\Gamma} \frac{K_l}{k}$$

where

$$\hat{\Gamma} = \left[ 1 + \frac{q}{k} \frac{(q - \hat{\Phi}k) \hat{C} - (q - k\hat{M}) \hat{B}}{k\hat{\Phi}(\hat{\Phi} + 2\hat{M}(\hat{C} - 1)) + k(\hat{B} + \hat{C} - 1)\hat{M}^2 + q\hat{B}\hat{M} - \hat{\Phi}q\hat{C}} \right]^{-1} < 1$$

\*\*\*\*\*PREUVE du point 5.4 (non donnée jusqu'à alors)

$$\begin{aligned}\min_r C(r) &= (c + \theta)q + (rk - \theta k\bar{\varphi} + K_l)F(r + c) + K_c \\ \rho + r(q - F(r + c)k) &= \theta q + (K_l - \bar{\varphi}k\theta)F(r + c) + K_c\end{aligned}$$

Then the solution is

$$\begin{aligned}C'(r) &= (rk - \theta k\bar{\varphi} + K_l)f(r + c) + F(r + c)k = 0 \\ \Leftrightarrow \tilde{r} + H(\tilde{r} + c) &= \bar{\varphi}\theta - \frac{K_l}{k}\end{aligned}$$

where  $H(z) = \frac{F(z)}{f(z)} \geq 0$  and increasing in  $z$  if  $F(z)$  is logconcave this implies

$$\tilde{r} < \bar{\varphi}\theta - \frac{K_l}{k}$$

And

$$\begin{aligned}\tilde{\rho} &= \left( (1 - \bar{\varphi})\theta + \frac{K_l}{k} \right) q + K_c + H(\tilde{r} + c) [q - F(\tilde{r} + c)k] \\ \Rightarrow \tilde{\rho} &> \left( (1 - \bar{\varphi})\theta + \frac{K_l}{k} \right) q + K_c\end{aligned}$$