To mitigate or to adapt: how to deal with optimism, pessimism and strategic ambiguity?

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Abstract

In this paper, we analyze the effect of ambiguity and ambiguity attitudes on optimal mitigation and adaptation contributions when players hold ambiguous beliefs about their opponents' behavior and their preferences can be modeled using the Choquet expected utility with neo-additive capacities. We find that ambiguity attitudes affect the amount contributed to these two policies. When players invest exclusively in mitigation, pessimists contribute more than optimists. When players choose both mitigation and adaptation, pessimists contribute more to mitigation, whereas optimists favor adaptation. Therefore, our results prove a dependence between equilibrium allocations and income distribution in presence of ambiguity. This dependence disappears once ambiguity vanishes. We investigate also the effect of two standard environmental policy instruments: standards and taxes, on mitigation policy. We find that in presence of ambiguity, the introduction of a binding standard does not systematically guarantee an increase in the total contribution to the mitigation policy. For the introduction of a tax, we find that an increase in the tax rate results in an increase in total mitigation and therefore a decrease in the private consumption. For small degrees of ambiguity, the optimal tax rate increases in the collective degree of optimism.

Keywords: Climate change; Mitigation; Adaptation; Ambiguity; pessimism, optimism, Choquet expected utility.

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1 Introduction

As never before, it is now widely believed that our planet is threatened by serious climatic disasters that affect significantly the quality of life of human beings. Indeed, we hear more often about environmental policies, collective ambitious efforts in order to combat climate change such as the Paris agreement, more public investment in research and development of new energetic technologies. Despite all the efforts made to reduce these environmental risks, individuals are confronted with a high degree of ambiguity that significantly reduces the effectiveness of such policies.

Heal and Milner (2013) have identified two types of ambiguity. On the one hand there is the scientific ambiguity which arises from our imperfect understanding of the evolution of the climate system. It is true that we know that climate change is happening. Nevertheless, a multitude of possible scenarios have been proposed to quantify these changes and their predictions differ drastically. A good example that can illustrate the imprecise aspect of these predictions is that given by the IPCC[1] (2014), where based on four different scenarios, the estimations have revealed distinct climate sensitivities and socioeconomic patterns. Therefore, regarding an increase in the overall temperature over the 21st century, predictions vary considerably between +0.3 and + 4.8°C.

On the other hand, there is the strategic ambiguity, which will be the focus of this paper. This latter arises from our lack of knowledge of how the economic agents will react to climate change. For instance, in spite of the commitment expressed by several countries to reduce their greenhouse gas emissions, following the Paris agreement, member countries are still not sure whether this commitment will be honored or not. Therefore they should make decisions in terms of environmental policies while holding ambiguous beliefs about the others' behavior. In view of this strategic ambiguity between economic agents, is the expected utility framework suitable to model their preferences? The well-known example of Ellsberg (1961) has clearly proved, using different experiments, that individuals present aversion towards situations in which probabilities are not perfectly known. Therefore we argue that non expected-utility frameworks are more suitable to be applied to the problem of climate change. Fortunately, there are considerable theoretical advances in the field of decision theory taking into account ambiguity (for a survey see Etner et al. (2012)).

Recently, some contributions have thus started applying non-expected utility frameworks to understand the relation between ambiguity and the problem of climate change such as Lange and Treich (2008), Millner et al. (2013), Lemoine and Traeger (2016), Berger et al. (2017) and Etner et al. (2020). However, we depart from these contributions in many dimensions since all of them focus on scientific ambiguity or what some authors, like Berger el al. (2017) or Heal and Millner (2014), call model uncertainty. In this paper, we focus on strategic instead of scientific ambiguity. In particular, we consider the case where players should take decisions while holding ambiguous beliefs about their opponents' behavior.

Moreover we grant a special attention to attitudes towards ambiguity. Indeed we believe that in

[1] IPCC: Inter-governmental Panel of experts on Climate Change.

presence of ambiguity, individuals will be heterogenous. Some of them are optimists and overweight the probability that their rivals are fully engaged in actions to fight against climate change. Some others are pessimists and overweight the probability that their rivals will make little or no efforts.

By focusing on both mitigation and adaptation as available environmental policies to improve the climatic system, and assuming that preferences under strategic ambiguity can be modeled using the Choquet Expected utility developed by Schmeidler (1989), we aim at studying the effect of ambiguity and attitudes towards ambiguity (optimism and pessimism) on the equilibrium contributions to both policies. We aim also at providing some policy recommendation in presence of ambiguity and agent heterogeneity.

1.1 Framework and results

We consider an OLG model where individuals can invest in two environmental policies: mitigation and adaptation. Individuals' preferences and beliefs are described by the Choquet expected utility representation developed by Schmeildler (1989) with neo-additive capacities as defined by Chateneauf et al. (2007). In the first period, individuals can split their endowment between personal savings and mitigation that benefits all players. In the second period their returns from savings will be used for consumption and adaptation. Ambiguity arises since players decide about their equilibrium contributions to mitigation policy while holding ambiguous beliefs about others' behavior. We consider two scenarios. One in which marginal costs of adaptation are constant. In this case, either mitigation or adaptation is exclusive optimal policy. The second scenario is the one in which marginal costs of adaptation are variable, then mitigation and adaptation are adopted simultaneously.

We find that ambiguity attitudes affect the amount contributed to these two policies. When all players contribute to mitigation, pessimists contribute more than optimists. Intuitively, pessimists overweight the possibility of 0-contributions on the side of their opponents. Since contributions are strategic substitutes, this leads to an increase in their equilibrium level of mitigation. In contrast, when adaptation is the optimal strategy, equilibrium allocations do not depend on the opponents' strategies since there is no ambiguity. When marginal costs of adaptation are variable, pessimists contribute more than optimists to mitigation, whereas optimists favor adaptation. Indeed, since pessimists spend a larger portion of their revenue for mitigation, they have less income available for adaptation in the second period. Our results prove also a dependence between equilibrium allocations and income distribution in presence of ambiguity. This dependence disappears once ambiguity vanishes.

In this paper, we are also interested in determining the optimal environmental policies in presence of ambiguity and agents' heterogeneity. Indeed, although standard environmental policy instruments such as taxes, subsidies, quotas, etc. are well understood in the context of certainty/risk, the study of these policies in the context of ambiguity is a novel area. We study, in the frame of this paper, the effect of introducing a standard on the amount contributed to mitigation policy and a tax on private consumption.

We find that the introduction of a binding standard is not always sufficient to increase contributions to mitigation in presence of agents' heterogeneity. Indeed, when the standard is binding only for optimists, pessimists will revise their beliefs about the worst-case scenario. This revision will be manifested in attributing a strictly positive weight to the optimists contributing m_s instead of 0. Hence, pessimists' contributions go down and consequently total mitigation. It it only when the standard becomes sufficiently high, that total mitigation increases.

For the introduction of a tax, we find that an increase in the tax rate results in an increase in total mitigation and therefore a decrease in the private consumption. Intuitively, when consumption becomes more expensive because of the tax, individuals will be less willing to invest in the "polluting" good and therefore their contribution to the "clean good" (mitigation) will increase. For small degrees of ambiguity, the optimal tax increases in the collective degree of optimism.

1.2 Organization of the paper

The paper is organized as follows. Section 2 is dedicated to a review of the existing literature. In section 3 we describe the economy, explain how we model strategic ambiguity and we discuss the equilibrium. Section 4 analyzes the game. We start by the case where marginal costs of adaptation are constant. The model is extended in a second step to a more realistic assumption where marginal costs of adaptation depend on total mitigation and initial environmental quality. In a third subsection, we study the effect of standards and taxes on equilibrium allocations. Finally, section 5 concludes.

2 Related literature

Our paper is related to studies focusing on (i) optimal environmental policies to fight against climate change, (ii) games with ambiguity, and (iii) climate change and ambiguity. We discuss each of these areas of literature below.

2.1 Climate change, mitigation and adaptation

Global climate change is a change in the long-term weather patterns that characterize the regions of the world. The impacts of climate change are already detected in different forms, from rising sea levels to changing weather patterns to shrinking ice sheets. The literature shows that climate change is a major issue that threatens our welfare and for which we must all mobilize by implementing various environmental policies such as mitigation and adaptation. The major aim of these policies is to substantially reduce the amount of green house gases (GHG) released into the atmosphere, see Venkataraman et al. (2012).

Heal and Kristrom (2002) defined mitigation as actions that reduce the flow of greenhouse gases into the atmosphere and so, change the probability distribution over future climate states. Adaptation refers to investment into processes, practices, or structures which moderates climate change damages and reduce the vulnerability of communities, regions, or countries to such environmental risks. (Buob and Stephan, 2011; Parry et al., 2007).

In the climate change debate, an overwhelming amount of analysis has focused on mitigation policies rather than adaptation, However IPCC's [2]latest recommendations and conclusions from international debates on climate change highlight that an effective climate policy could be a mix of mitigation and adaptation actions (Parry et al., 2007).

Over time, some papers have realized the relevance of the mixed policy and they concentrate on mitigation and adaptation simultaneously e.g., Kane and Yohe (2000), McKibbin and Wilcoxen (2003), Tol (2005), Ingham et al. (2006). However these papers have been descriptive and none of them have adressed the problem in a theoretical context. To the best of our knowledge, Buob and Stephan (2011) is the first study to consider adaptation and mitigation as policy responses to global climate change within a game-theoretic framework. We follow this study in order to explore the effects of mitigation and adaptation policies on climate change in the presence of strategic ambiguity.

2.2 Ambiguity in games

An ambiguous situation is a situation of uncertainty where the decision maker's belief is not precise enough to be represented by a single probability distribution. The decision theory literature has long recognized that the expected utility might be unsuitable to represent ambiguous beliefs. For that, alternative functionals to expected utility have been introduced.

The literature in game theory comprises three approaches which model ambiguity in games: Objective, Contextual and Subjective ambiguity approaches, see the review by Beauchêne (2014). The first approach extends the strategy set by allowing players to use objectively ambiguous randomization devices, Bade (2011), Riedel and Sass (2014). The contextual ambiguity approach considers games with incomplete information, in which probabilities over types are unknown, see e.g., Hanany, Klibanoff and Mukerji (2016). Finally, the Subjective approach assumes that players perceive strategic ambiguity, even though in equilibrium players' strategy choice is not ambiguous, and defines an equilibrium in beliefs, see Dow and Werlang (1994), Lo (1996), Marinacci (2000), Eichberger and Kelsey (2000, 2002). In the frame of this work, we will concentrate on strategic ambiguity.

Several experimental papers have shown that deviations of observed behavior from Nash equilibrium can be explained by strategic ambiguity, see Colman and Pulford (2007), Di Mauro and

[2]IPCC: Inter-governmental Panel of experts on Climate Change.

Castro (2008), Eichberger, Kelsey and Schipper (2008), Kelsey and Le Roux (2015, 2016) and the analysis in Eichberger and Kelsey (2011).

The approach taken by Eichberger and Kelsey (2000) has been applied to games of private provision of public goods, Eichberger and Kelsey (2002), as well as to models of oligopolies and to coordination games, Fontitni (2005), Eichberger, Kelsey and Schipper (2009). In coordination games, ambiguity aversion or ambiguity loving behavior can serve as an equilibrium selection device. Additionally, it was found that ambiguity will increase/decrease the equilibrium strategy in games with strategic complements/substitutes and positive externalities. These effects are reversed in games with negative externalities, see also Schipper (2005). In games of strategic substitutes with externalities such as public good provision and Cournot duopoly, an increase in ambiguity combined with pessimism can bring the equilibrium allocation closer to Pareto-optimality, while an increase in optimism increases the player's own payoff, but results in Pareto-inferior allocations, see Eichberger, Kelsey and Schipper (2009) and Kelsey and Le Roux (2015).

These findings suggest that ambiguity and heterogeneous attitudes towards ambiguity (pessimism/optimism) might be relevant to explain decisions makers' contribution to environmental policies. Despite the vast literature on climate change and optimal policies, to our knowledge this issue has not been considered so far.

2.3 Climate change and ambiguity

The literature that we have just presented above has separately studied the issues related to climate change and ambiguity in games. Nevertheless, a growing number of researchers recognize that ambiguity is intrinsic to climate change. Heal and Millner (2013) has identified two sources of uncertainty[3] related to climate change. On the one hand, there is the scientific uncertainty which arises from lack of knowledge about some environmental events. It is true that we know that the climate is changing, but not precisely how fast or in what ways. Such incomplete knowledge can significantly reduce the efficiency of policies undertaken. On the other hand, there is the socioeconomic uncertainty or also called strategic uncertainty. This latter arises from our incomplete understanding of how individuals will react to climate change.

Some recent papers have thus started applying non-expected utility frameworks to the problem of climate change. These contributions include: Lange and Treich (2008), who study ambiguity in a model of environmental pollution and find that ambiguity aversion pushes individuals to reduce their emissions and to be more cautious toward pollution. Lemoine and Traeger (2012); Millner et al. (2013) who propose numerical models based on stochastic variants of the DICE model under ambiguity aversion. Berger et al. (2016) who study the impact of ambiguity and ambiguity aversion on optimal mitigation level and quantify this impact using a Dynamic Integrated Climate Economy (DICE) model. And finally Etner et al. (2020) who study the effect of ambiguity and

[3]In the rest of this paper, uncertainty and ambiguity are considered as synonym.

ambiguity attitude on optimal adaptation and mitigation decisions when the future environmental quality is ambiguous. The main difference between these two last papers and ours is that they focus on scientific ambiguity where the future environmental quality is ambiguous while we focus on strategic ambiguity where we assume that individuals hold ambiguous beliefs about their opponents' behavior.

3 An OLG-model with mitigation and adaptation

3.1 The economy

Consider an OLG model where in each period a mass of n consumers is born. Consumers live for two periods and receive y_i units in the first period of their life and no endowment in the second period. Young consumers can use their endowment to save S_i and to contribute to a mitigation policy m_i that benefits to all players. Since old consumers have no endowment in the second period of their life, the returns on savings determine their consumption and their contribution to the adaptation policy.

Formally, consumer i chooses a mitigation effort $m_{i,t}$ and savings $S_{i,t}$ with

$$y_{i,t} = m_{i,t} + S_{i,t}$$

Environmental quality[4] in the next period is given by

$$E_{t+1} = E_t + m_{i,t} + m_{j,t}$$

Consumers choose their mitigation efforts simultaneously. These choices become commonly known in the second period and will affect global environmental quality. Household i can use his accrued savings $(1+r)S_{i,t}$ to adapt to the new environmental quality by choosing an adaptation effort $a_{i,t+1}$ which has no strategic effects or for private consumption $c_{i,t+1}$. His budget constraint for period two is given by:

$$(1+r)S_{i,t} = c_{i,t+1} + h(a_{i,t+1}, E_{t+1})$$

[4]We follow Raffin and Seegmuller (2014), John and Pecchenino (1994), Jouvet et al. (2005) to describe the evolution of the environmental quality over time. However, we assume that the parameter reflecting the efficiency of mitigation is equal to one contrary to the cited papers.

Where h is the cost function of adaptation.

The perceived environmental quality taking into account adaptation is as follows:

$$e_{i,t+1} = E_t + M_t + a_{i,t+1}$$

3.2 Preferences and strategic ambiguity

Consider the game $\Gamma = \langle N, (X_i)(u_i) : 1 \leqslant i \leqslant n \rangle$ with finite pure strategy sets X_i for each player such as

$$\tilde{X}_i^1 = \{x_i = (m_i, S_i) \mid y_i = m_i + S_i\}$$
$$\tilde{X}_i^2 = \{x_i = (C_i, a_i) \mid (1+r)(y_i - m_i) = c_i + h(a_i)\}$$

The notation , x_{-i}, indicates a strategy combination for all players except i. The space of all such strategy profiles is denoted by X_{-i}.

We denote the payoff to player i from choosing their strategy x_i in $\tilde{X}_i^2$ when their opponents have chosen x_{-i} in $\tilde{X}_{-i}^2$ by $u_i(x_i, x_{-i})$. Assume that this payoff can be represented by a Cobb-Douglas utility function such that

$$u_i(x_i, x_{-i}) = C_i[E + m_i + m_{-i} + a_i] = C_i[E + M + a_i]$$

with $M = m_i + m_{-i}$ denoting total mitigation efforts.

We assume that player i makes decisions about m_i while holding ambiguous beliefs about their opponents' behavior. These ambiguous beliefs of player i on $\tilde{X}_{-i}$ can be modeled using a particular class of capacities called neo-additive capacities as axiomatized by Chateauneuf et al. (2007).

Definition 1 *Let δ and α be real numbers such that $0 \leqslant \delta \leqslant 1$ and $0 \leqslant \alpha \leqslant 1$. A neo-additive-capacity ν_i on $\tilde{X}_{-i}$ is defined by $\nu_i(\varnothing) = 0$, $\nu_i\left(\tilde{X}_{-i}\right) = 1$ and $\nu_i(A) = \delta_i\alpha_i + (1-\delta_i)\pi_i(A)$ for $\varnothing \subset_{\neq} A \subset_{\neq} \tilde{X}_{-i}$, where π_i is an additive probability distribution on $\tilde{X}_{-i}$.*

In this n-player game, we will only consider pure strategy equilibria, hence π_i assigns a probability 1 to a single strategy of the opponent.[5]

Definition 2 *The Choquet expected payoff with respect to the neo-additive capacity axiomatised by (Chateauneuf et al; 2007) is given by:*

$$\begin{aligned} V_i(x_i \mid \nu_i(. \mid \alpha_i, \delta_i, \pi_i)) &= \int u_i(x_i, x_{-i})\, dv \\ &= \delta_i\alpha_i \max_{x_{-i}\in\tilde{X}_{-i}} u_i(x_i, x_{-i}) + \delta_i(1-\alpha_i) \min_{x_{-i}\in\tilde{X}_{-i}} u_i(x_i, x_{-i}) \\ &\quad + (1-\delta_i)\, u_i(x_i, x_{-i}) \end{aligned}$$

[5] Since we do not consider mixed strategies we do not need to worry about the modelling of independent mixtures under ambiguity.

This expression is a weighted average of the highest payoff, the lowest payoff and the expected payoff for a given strategy x_i played by i. The response to ambiguity is partly optimistic represented by the weight α_i given to the best outcome and partly pessimistic represented by the weight $(1-\alpha_i)$ given to the worst outcome. These preferences are a special case of Choquet expected utility (CEU), see Chateauneuf et al. (2007).

Definition 3 *The support of the neo-additive capacity, $\nu(A) = \delta\alpha + (1-\delta)\,\pi(A)$, is defined by*[6] *$supp\,(\nu) = supp\,(\pi)$*

Define the best response correspondence of player i given that his/her beliefs are represented by a neo-additive capacity v_i by

$$\rho_i(\nu_i) = \arg\max_{x_i \in \tilde{X}_i} V_i\left(x_i \mid \nu_i\left(. \mid \alpha_i, \delta_i, \pi_i\right)\right).$$

The equilibrium under ambiguity is defined as follows, see Eichberger and Kelsey (2014):

Definition 4 *(Equilibrium under ambiguity) A vector of neo-additive capacities (ν_i^*, ν_{-i}^*) is an equilibrium under ambiguity (EUA) if for $i \in \{1,..,n\}$, $supp\,(\nu_i^*) \subseteq \rho_{-i}\left(v_{-i}^*\right)$.*

4 Analysis of the static game

We now proceed to analyze the mitigation and adaptation decisions. For the purposes of the analysis of the game, we assume that the perceived ambiguity is identical for the n players and so, $\delta_i = \delta\ \forall i \in \{1,..,n\}$

4.1 All players invest in mitigation

We first concentrate on an equilibrium of the static game in which all players invest in mitigation. Note that the payoff of a player when all players invest in mitigation can be rewritten in terms of only the mitigation efforts of the players:

$$V_i(m_i, m_{-i}) = (1+r)\,(y_i - m_i)\left[E + m_i + (1-\delta)m_{-i} + \alpha_i\delta\sum_{j\neq i} y_j\right]$$

The best response is given by:

$$\rho_i(m_{-i}) = \begin{cases} \frac{y_i - E - \alpha_i\delta\sum_{j\neq i} y_j - (1-\delta)m_{-i}}{2} & \text{if } m_{-i} \leq \frac{y_i - E - \alpha_i\delta\sum_{j\neq i} y_j}{1-\delta} \\ 0 & \text{else} \end{cases}$$

[6] For a justification of this definition and its relation to other support notions see Eichberger and Kelsey (2014).

Proposition 5 *The unique interior Nash equilibrium under ambiguity of the static game is given by:*

$$\begin{aligned} m_i^*(\delta,\alpha_i) &= \frac{[2\delta+n(1-\delta)]\,y_i-(1+\delta)E}{[1+\delta+n(1-\delta)]\,(1+\delta)} \\ &+\frac{[\delta(1-\delta)\sum_{k=1}^{n}\alpha_k-(1-\delta)]\sum_{j\neq i}y_j}{[1+\delta+n(1-\delta)]\,(1+\delta)}-\frac{[1+\delta+n(1-\delta)]\,\alpha_i\delta\sum_{j\neq i}y_j}{[1+\delta+n(1-\delta)]\,(1+\delta)} \end{aligned}$$

It satisfies the following properties

(i) $\frac{\partial m_i^*(\delta)}{\partial \alpha_i}<0$;

(ii) $\frac{\partial m_i^*(\delta)}{\partial E}<0$;

(iii) Total mitigation is given by $M^*(\delta)=\frac{\sum_{i=1}^{n}y_i-nE-\delta\sum_{i=1}^{n}\alpha_i\sum_{j\neq i}y_j}{1+\delta+n(1-\delta)}$ with $\frac{\partial M^*}{\partial\sum_{i=1}^{n}\alpha_i}<0$

Note that the contribution to mitigation depends negatively on the individual degree of optimism. Hence, we expect from this expression that an extreme pessimist whose $\alpha_p=0$, will contribute more than an extreme optimist whose $\alpha_o=1$. Intuitively, the pessimist attributes a strictly positive weight δ to the optimist choosing 0 mitigation. Since mitigation efforts are substitutes, he compensates by increasing his own mitigation effort. A symmetric argument holds for the optimist. Eichberger, Kelsey and Schipper (2009) discuss the effect of optimism and pessimism in games with strategic substitutes and complements.

Furthermore, the pessimist's mitigation effort increases in the degree of ambiguity, whereas that of the optimist decreases. However, the increase in the pessimist's contribution is not sufficient to compensate for the reduction in that of the optimist. It follows that total mitigation in the Nash equilibrium under ambiguity decreases with perceived degree of ambiguity and is maximal when $\delta=0$. Note that total mitigation is also decreasing in the collective degree of optimism. This negative relation can be explained by the fact that when the collective degree of optimism increases in the society, individuals will be less willing to contribute to environmental policies given their optimistic beliefs leading to a decrease in the total mitigation. However, the effect of a variation of the exogenous degree of ambiguity on total mitigation depends on the value of $\sum_{i=1}^{n}\alpha_i$. Ambiguity affects positively total mitigation only if the collective degree of optimism in the society is below $\frac{(n-1)\sum_{i=1}^{n}y_i+n(1-n)E}{(n+1)\sum_{j\neq i}y_j}$.

Interestingly, total mitigation depends not just on the total income in the society, but also on the income distribution. Note that this dependence on the income distribution disappears once ambiguity vanishes $\delta=0$

Until now, we provided a characterization of an equilibrium where all players invest in mitigation. However, asymmetric equilibria, where one player has interest to invest in mitigation and the others do nothing, exist as well.

4.2 All players invest in adaptation

Here we consider the case in which all players invest in adaptation a_i^*.Then $m_i^* = m_{-i}^* = 0$ and player i maximizes :

$$U_i(a_i) = [(1+r)y_i - \phi a_i]\,[E + a_i]$$

Proposition 6 *The equilibrium adaptation efforts and consumption are given by:*

$$a_i^* = \frac{(1+r)y_i - \phi E}{2\phi},\;\; c_i^* = \frac{(1+r)y_i + \phi E}{2}$$

Observe that in this case, individual adaptation is negatively related to the initial environmental quality E, in contrast to consumption, for which a positive relationship exists. Higher values of E lead to less adaptation. In fact, for any $E \geqslant E = \frac{(1+r)y_i}{2\phi}$, None of the players, has interest to invest in adaptation policy since the marginal utility of contributing to adaptation is too low as compared to that of consumption. We can also note that as benefits of adaptation are private to each player, individual adaptation, consumption and environmental quality do not depend on the opponent's strategies.

4.3 Variable marginal costs of adaptation

In what follows, we will assume that contributions to mitigation chosen by the players in period 1 will ameliorate the global environmental quality which in turn reduces the cost of adaptation policy. Similarly a higher initial environmental quality can reduce vulnerability to climate change and as a consequence lower the cost of investment in adaptation.

We use the shape of the cost function introduced by Buob and Stephan (2011) which is as follows

$$h(a_i) = \frac{\phi}{E+M} a_i$$

The second period utility function of the player can be written as follows:

$$U_i(m_i, a_i(m_i, m_{-i}), m_{-i}) = \left[(1+r)\,(y_i - m_i) - \frac{\phi}{E+M} a_i\right][E + M + a_i(m_i, m_{-i})]$$

Proposition 7 *If* $h(a_i) = \frac{\phi}{E+M} a_i$ *, then*

$$a_i^*(m_i, m_{-i}) = \begin{cases} \frac{(E+M)}{2\phi}\left[(1+r)(y_i - m_i) - \phi\right] & \text{if } (1+r)(y_i - m_i) - \phi > 0 \\ 0 & \text{else} \end{cases}$$

Assume that player i entertains a capacity with $\pi_i(m_{-i}) = 1$ for some $m_{-i} \in S_{-i}$. Then i will maximize the following Choquet expected utility function:

$$V_i(m_i, m_{-i}) = \frac{1}{4\phi}\left[(1+r)(y_i - m_i) + \phi\right]^2 \left[E + m_i + (1-\delta)m_{-i} + \delta\alpha_i \sum_{j \neq i} y_j\right]$$

We obtain the best-response for the mitigation policy as:

$$\rho_i(m_{-i}) = \begin{cases} \frac{(1+r)y_i + \phi - 2(1+r)E - 2(1-\delta)(1+r)m_{-i} - 2(1+r)\alpha_i\delta\sum_{j\neq i} y_j}{3(1+r)} & \text{if } m_{-i} \leq \frac{(1+r)y_i + \phi - 2(1+r)\left[E - \alpha_i\delta\sum_{j\neq i} y_j\right]}{2(1-\delta)(1+r)} \\ 0 & \text{else} \end{cases}$$

Proposition 8 *Mitigation and adaptation allocations are given by:*

$$\begin{aligned} m_i^*\left(\delta, \alpha_i, \sum_{j\neq i}\alpha_j\right) &= \frac{\left(3 + 4\sum_{j\neq i}\alpha_j\delta(1-\delta)\right)y_i}{\left[3-2(1-\delta)\right]\left[3+2(1-\delta)\right]} + \frac{(1-2\delta)\phi}{(1+r)\left[3-2(1-\delta)\right]\left[3+2(1-\delta)\right]} \\ &\quad - \frac{2(1+r)(1+2\delta)E}{(1+r)\left[3-2(1-\delta)\right]\left[3+2(1-\delta)\right]} - \frac{2(1+r)\sum_{j\neq i} y_j\left[(1-\delta) + 3\alpha_i\delta\right]}{(1+r)\left[3-2(1-\delta)\right]\left[3+2(1-\delta)\right]} \end{aligned}$$

$$\begin{aligned} a_i^*\left(\delta, \alpha_i, \sum_{j\neq i}\alpha_j\right) &= \frac{(E+M)}{2\phi} \\ &\quad \left[\frac{\left[6 - 4(1-\delta)\left(1-\delta\left(1-\sum_{j\neq i}\alpha_j\right)\right)\right](1+r)y_i}{\left[3-2(1-\delta)\right]\left[3+2(1-\delta)\right]} + \frac{2(1+r)\left[(1-\delta)+3\alpha_i\delta\right]\sum_{j\neq i} y_j}{\left[3-2(1-\delta)\right]\left[3+2(1-\delta)\right]}\right. \\ &\quad \left. + \frac{2(1+r)(1+2\delta)E}{\left[3-2(1-\delta)\right]\left[3+2(1-\delta)\right]} - \frac{2\left(3+3\delta-2\delta^2\right)\phi}{\left[3-2(1-\delta)\right]\left[3+2(1-\delta)\right]}\right] \end{aligned}$$

They satisfy the following properties:

(*i*) for $\delta \in (0,1]$, $\frac{\partial m_i^*}{\partial \alpha_i} = \frac{-6\delta\sum_{j\neq i} y_j}{[3-2(1-\delta)][3+2(1-\delta)]} < 0$

(*ii*) for $\delta \in (0,1]$ $\frac{\partial a_i}{\partial \alpha_i} = \frac{(E+M)}{\phi}\frac{3\delta(1+r)\sum_{j\neq i} y_j}{[3-2(1-\delta)][3+2(1-\delta)]} > 0$;

Note that in the presence of ambiguity, ($\delta > 0$), an increase in optimism reduces the level of the equilibrium contributions to mitigation policy. This finding is in line with the results when marginal costs of adaptation are constant. Intuitively, when the players overweight the probability that their rivals will make little or no efforts to fight against climate change, they assign a strictly positive weight δ to the worst-case scenario where the opponents contribute nothing to mitigation. As mitigation efforts are considered as perfect substitutes, pessimists compensate by increasing their own mitigation. Since pessimists spend a larger portion of their revenue for mitigation, they have less income available for adaptation in the second period.

People with optimistic beliefs will give more weight to the possibility that other players will spend their entire income to the mitigation policy. With such beliefs, optimists will contribute less than pessimists to mitigation in period 1. As they will have more disposable income in Period 2. Therefore, they will contribute more to the adaptation policy in order to improve their individual environmental quality.

We contribute to the literature by giving some insights about the complementarity vs substitutability between mitigation and adaptation strategies. Indeed, under strategic ambiguity, these two climate policies are considered as substitutes in the sense that strengthening one type of policy will weaken the other.

Note also that pessimists are more altruistic than optimists in the sense that they contribute more to the public good (mitigation) than to the private good (adaptation). Our result is in line with Ingham et al. (2013) who examine a variety of economic models with mitigation and adaptation and prove that these policies are most likely to be substitute.

4.4 Introduction of standards and taxes

In this subsection, we study the effect of the introduction of standards and taxes on the equilibrium contributions. In order to simplify calculations, we assume that marginal costs of adaptation are constant and players will invest only in mitigation policy.

4.4.1 Introduction of a standard

For the purposes of the analysis, we will assume that the n consumers present in the economy can be classsified into two representative agents with extreme attitudes to ambiguity: An optimist ($\alpha_o = 1$) and a pessimist ($\alpha_p = 0$)

Let's assume that the government will implement a standard on the amount contributed to mitigation policy by the two representative players. This standard imposes a lower bound on the contributions which should be higher than m_s.

The Choquet expected utility of an optimist is given by[7]:

[7] To simplify calculations, we will assume that $y = y_o = y_p$

$$V_o(m_o, m_p) = (y - m_o)\left[\psi(E) + m_o + (1-\delta)m_p + \delta y\right] \tag{1}$$

The Choquet expected utility of a pessimist is given by:

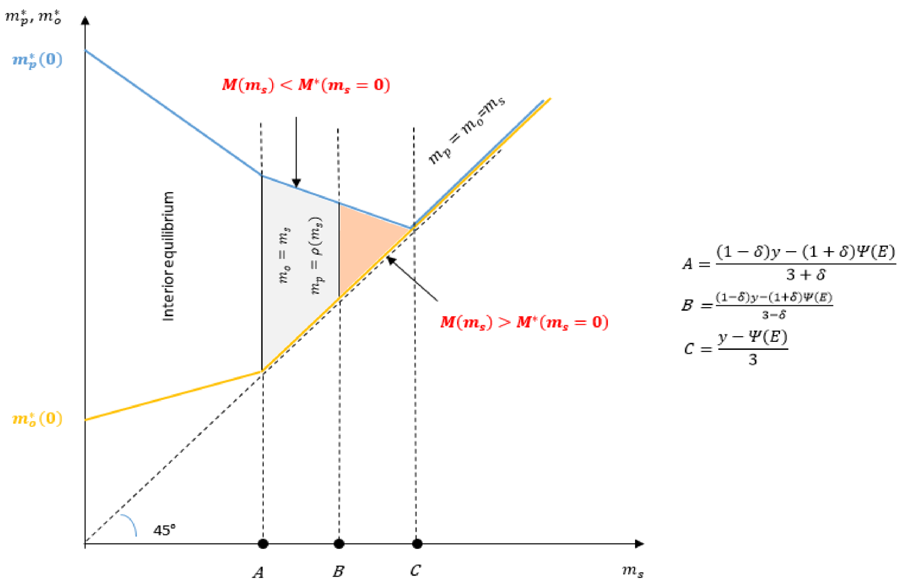
$$V_p(m_o, m_p) = (y - m_p)\left[\psi(E) + m_p + (1-\delta)m_o + \delta m_s\right] \tag{2}$$

Proposition 9 *The equilibrium contributions of the pessimist and the optimist under the presence of a standard are given by:*

$$m_o^{s*} = \begin{cases} \frac{(1-\delta)y-(1+\delta)\psi(E)+\delta(1-\delta)m_s}{(3-\delta)(1+\delta)} & \text{if } m_s \leq \frac{(1-\delta)y-(1+\delta)\psi(E)}{3+\delta} \\ m_s & \text{else} \end{cases}$$

$$m_p^{s*} = \begin{cases} \frac{\left(1+2\delta-\delta^2\right)y-(1+\delta)\psi(E)-2\delta m_s}{(3-\delta)(1+\delta)} & \text{if } m_s \leq \frac{(1-\delta)y-(1+\delta)\psi(E)}{3+\delta} \\ \rho_p\left(m_s\right) = \frac{y-\psi(E)-m_s}{2} & \text{if } m_s \in \left[\frac{(1-\delta)y-(1+\delta)\psi(E)}{3+\delta}, \frac{y-\psi(E)}{3}\right] \\ m_s & \text{else} \end{cases}$$

Let's recapitulate the diferent levels of contributions by the following graph.



In the absence of a standard imposed by the government, pessimist's mitigation efforts exceed that of the optimist as shown in section (4). Intuitively, the pessimist attributes a strictly positive weight to the optimist choosing 0 mitigation and therefore he compensates by increasing his own mitigation effort.

With the introduction of a standard $(m_s \geqslant 0)$, the pessimist will revise his belief about the the worst-case scenario. This revision will be manifested in attributing a strictly positive weight to the opponent contributing m_s instead of 0. Therefore, the pessimist will reduce his mitigation efforts. The optimist' s contribution increases with m_s

When the standard reaches a certain value $\left(m_s \geqslant \frac{(1-\delta)y-(1+\delta)\psi(E)}{3+\delta}\right)$, the optimist's equilibrium contribution becomes lower than m_s and so the optimist will be forced to contribute m_s. The pessimist will play a best-response to the strategy of his opponent (m_s), given by $\rho_p(m_s) = \frac{y-\psi(E)-m_s}{2}$. When the standard is sufficiently high $\left(m_s \geqslant \frac{y-\psi(E)}{3}\right)$, even the pessimist's equilibrium contribution becomes lower than m_s. His contribution will thus be given by m_s.

When the standard is binding only for the optimist $\left(m_s \geqslant \frac{(1-\delta)y-(1+\delta)\psi(E)}{3+\delta}\right)$, the pessimist will revise his beliefs about the contribution of his opponent and thus reduces his own mitigation efforts. Despite the increase in the optimist's contribution, the reduction in the pessimist's contribution is higher. This results in an overall decrease in the total amount contributed to mitigation policy. This finding is a contradiction to the desired effect of the social planner. It is only when a certain value of the standard is reached $\left(m_s = \frac{(1-\delta)y-(1+\delta)\psi(E)}{3-\delta}\right)$ that the increase in the optimist's contribution compensates the reduction in the contribution of the pessimist to generate an increase in the total mitigation.

4.4.2 Introduction of taxes

In this part, we study the effect of a tax on private consumption in equilibrium allocations.

Each player i will maximize the following Choquet expected utility function:

$$V_i(m_i, m_{-i}) = \frac{(1+r)}{(1+\tau)}(y_i + t_i - m_i)\left[\psi(E) + m_i + (1-\delta)m_{-i} + \alpha_i\delta\sum_{j\neq i}(y_j + t_j)\right]$$

Proposition 10 *Suppose that the policy maker imposes a tax on private consumption with τ the tax rate*[8]*. Then:*

(*i*) Total mitigation is given by:

$$M^*(\tau) = \frac{\left[1 + \left[1 - \sum_{i=1}^n \alpha_i\delta\frac{(n-1)}{n}\right]\tau\right]y - nE - \sum_{i=1}^n \alpha_i\delta\sum_{j\neq i} y_j}{(n+1+\delta(n-1)) + \left[1 - \sum_{i=1}^n \alpha_i\delta\frac{(n-1)}{n}\right]}$$

(*ii*) Total consumption is given by

$$C^*(\tau) = (1+r)\left[\frac{\left[(n+1+\delta(n-1)) - \tau - \sum_{i=1}^n \alpha_i\delta\frac{(n-1)}{n}(1-\tau)\right]y + nE + \sum_{i=1}^n \alpha_i\delta\sum_{j\neq i} y_j}{(n+1+\delta(n-1)) + \left[1 - \sum_{i=1}^n \alpha_i\delta\frac{(n-1)}{n}\right]}\right]$$

[8] Here we will assume that $t_i = t$, for all i, the lump sum transfers do not depend on income or on optimism/pessimism

They satisfy:

(i) $\frac{\partial M^*(\tau)}{\partial \tau} > 0$

(ii) $\frac{\partial C^*(\tau)}{\partial \tau} < 0$

As expected, an increase in the tax rate results in an increase in total mitigation and therefore a decrease in total consumption. Intuitively, when consumption becomes more expensive because of the tax, individuals will be less willing to invest in the "polluting" good and therefore their contribution to the "clean good" (mitigation) will increase.

In order to go further in our analysis, we will determine the Pareto-optimal tax that should be imposed by the social planner. Let us, at first, present the Pareto-optimal mitigation under the condition that the utilities of the n players are weighted equally by the social planner.

To simplify calculations, we assume also that $y_i = \frac{y}{n}$ for $\forall i \in \{1, .., n\}$.

Proposition 11 *If the social planner will maximize the following Choquet expected utility function:*

$$V^S(m_i, m_{-i}) = (1+r)\sum_{i=1}^{n}(y_i - m_i)\left[E + m_i + (1-\delta)m_{-i} + \alpha_i\delta\sum_{j\neq i} y_j\right]$$

The total Pareto-optimal mitigation will be given by:

$$M^S(\delta) = \frac{[1 + (1-\delta)(n-1)]\,y - nE - \delta\sum_{i=1}^{n}\alpha_i\sum_{j\neq i} y_j}{2[\delta + n(1-\delta)]}$$

Once we have identified the Pareto-optimal mitigation, we can determine the Pareto-optimal tax by equalizing Pareto-optimal mitigation to equilibrium mitigation under taxes.

Proposition 12 *The Pareto-optimal tax rate is given by:*

$$\tau^S = \frac{\left[\frac{(n-1)(1+\delta)}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right]} + 1\right]y - \left[1 - \frac{(n-1)(1-3\delta)}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right][n-\delta(n-1)]}\right]\left[nE + \delta\sum_{i=1}^{n}\alpha_i\sum_{j\neq i} y_j\right]}{2y}$$

It satisfies[9]

- $\frac{\partial \tau^S}{\partial \sum_{i=1}^{n}\alpha_i} > 0$ for $n > 4$,any $\delta \in \left(0, \frac{1}{3}\right)$ such that $1 + \delta > \frac{n}{n-1}$;

Proposition (12) allows us to underscore the effect of agents' heterogeneity in terms of ambiguity attitude (pessimism/ optimism) on tax rate: The optimal tax rate increases in the collective degree of optimism when the degree of ambiguity is sufficiently small, $\delta \in \left(0, \frac{1}{3}\right)$.

[9]In order to simplify calculations, we assume that $y_j = const$

5 Conclusion

This paper aims at studying the impact of strategic ambiguity and ambiguity attitudes on optimal mitigation and adaptation contributions. We use the Choquet Expected utility developed by Schmeildler (1989) with neo-additive capacities to model preferences when players hold ambiguous beliefs about their opponents' behavior.

In presence of ambiguity ($\delta \geq 0$), we show that optimism induces the player to be more ambitious about the contributions of their opponents to climate policy. Players with pessimistic beliefs attribute more weight to the worst-case scenario where others make little or no efforts to fight against climate change. This is why they are more willing to contribute to the public good (mitigation policy) than to the private good (adaptation policy). We can therefore conclude that pessimism could be good for environment.

Our paper points to the importance of ambiguity attitudes (pessimism/optimism) in the process of designing environmental policy instruments such as taxes and standards. Holding information about agents' heterogeneity is crucial to choose better the optimal environmental instruments.

In this paper, we restrict attention to a static game, a dynamic framework where the share of pessimists and optimists change in response to past performance should be investigated soon in the future. It would be also interesting to test our results in an experimental framework using a lab experiment.

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7 Appendices

Proof of Proposition 5:

$$V_i(m_i, m_{-i}) = (1+r)\,(y_i - m_i)\left[\psi(E) + m_i + (1-\delta)m_{-i} + \alpha_i\delta\sum_{j\neq i} y_j\right]$$

$$\frac{\partial V_i}{\partial m_i} = (1+r)\left[y_i - \psi(E) - 2m_i - (1-\delta)m_{-i} - \alpha_i\delta\sum_{j\neq i} y_j\right]$$

$$y_i - \psi(E) - (1-\delta)M - \alpha_i\delta\sum_{j\neq i} y_j = (1+\delta)m_i$$

$$\sum_{i=1}^{n} m_i = \frac{\sum_{i=1}^{n} y_i - n\psi(E) - (1-\delta)nM - \delta\sum_{i=1}^{n}\alpha_i\sum_{j\neq i} y_j}{(1+\delta)}$$

Total mitigation is given by:

Individual mitigation is given by the following expression:

$$m_i = \frac{y_i - \psi(E) - (1-\delta)M - \alpha_i\delta\sum_{j\neq i} y_j}{1+\delta}$$

We replace total mitigation by its expression, we get the following result:

$$\begin{aligned} m_i^*(\delta,\alpha_i) \quad = \quad & \frac{[2\delta + n(1-\delta)]\,y_i - (1+\delta)\psi(E)}{[1+\delta+n(1-\delta)]\,(1+\delta)} \\ & + \frac{[\delta(1-\delta)\sum_{k=1}^{n}\alpha_k - (1-\delta)]\sum_{j\neq i} y_j}{[1+\delta+n(1-\delta)]\,(1+\delta)} - \frac{[1+\delta+n(1-\delta)]\,\alpha_i\delta\sum_{j\neq i} y_j}{[1+\delta+n(1-\delta)]\,(1+\delta)} \end{aligned}$$

Proof of Proposition 6

$$U_i(a_i) = [(1+r)y_i - \phi a_i]\,[E + a_i]$$

$$\frac{\partial U_i}{\partial a_i} = -\phi E - 2\phi a_i + (1+r)y_i$$

$$\begin{aligned} a_i^* &= \frac{(1+r)y_i - \phi E}{2\phi} \\ c_i^* &= \frac{(1+r)y_i + \phi E}{2} \end{aligned}$$

Proof of Proposition 7:

$$U_i(a_i, m_i) = \left[(1+r)\left(y_i - m_i\right) - \frac{\phi}{E+M} a_i\right]\left[E + M + a_i\right]$$

$$\frac{\partial U_i}{\partial a_i} = -\frac{\phi}{E+M}\left[E+M+a_i\right] + \left[(1+r)\left(y_i - m_i\right) - \frac{\phi}{E+M} a_i\right]$$

$$\frac{\partial U_i}{\partial a_i} = -\frac{\phi}{E+M}\left[E+M+a_i\right] + \left[(1+r)\left(y_i - m_i\right) - \frac{\phi}{E+M} a_i\right] = 0$$

$$(1+r)\left(y_i - m_i\right) - \phi = \frac{2\phi}{E+M} a_i$$

$$a_i^* = \frac{(E+M)}{2\phi}\left[(1+r)\left(y_i - m_i\right) - \phi\right]$$

Proof of Proposition 8:

The Choquet expected utility can be written as

$$\begin{aligned}
V_i\left(m_i, m_{-i}\right) &= C_i\left[E + m_i + (1-\delta)\, m_{-i} + \delta\alpha_i \sum_{j\neq i} y_j + \hat{a}_i\left(m_i, m_{-i}\right)\right] \\
V_i\left(m_i, m_{-i}\right) &= \left[(1+r)\left(y_i - m_i\right) + \phi\right] \\
&\quad \left(E + m_i + (1-\delta)\, m_{-i} + \delta\alpha_i \sum_{j\neq i} y_j + \frac{E + m_i + (1-\delta)\, m_{-i} + \delta\alpha_i \sum_{j\neq i} y_j}{2\phi}\left[(1+r)\left(y_i - m_i\right)\right.\right. \\
V_i\left(m_i, m_{-i}\right) &= \left[\frac{(1+r)\left(y_i - m_i\right) + \phi}{2}\right]\left(\left(E + m_i + (1-\delta)\, m_{-i} + \delta\alpha_i \sum_{j\neq i} y_j\right)\left(\frac{\left[(1+r)\left(y_i - m_i\right) + \phi\right]}{2\phi}\right)\right)
\end{aligned}$$

$$V_i\left(m_i, m_{-i}\right) = \frac{1}{4\phi}\left[(1+r)\left(y_i - m_i\right) + \phi\right]^2\left[E + m_i + (1-\delta)m_{-i} + \alpha_i\delta\sum_{j\neq i} y_j\right]$$

The best response of a player i

$$\frac{\partial V_i}{\partial m_i} = \frac{1}{4\phi}\left[-2\left(1+r\right)\left[(1+r)\left(y_i - m_i\right) + \phi\right]\left[E + m_i + (1-\delta)m_{-i} + \alpha_i\delta\sum_{j\neq i} y_j\right] + \left[(1+r)\left(y_i - m_i\right) + \phi\right]^2\right]$$

$$(1+r)y_i-(1+r)m_i+\phi-2\,(1+r)\left[E+m_i+(1-\delta)m_{-i}+\alpha_i\delta\sum_{j\neq i}y_j\right]=0$$

$$\rho_i\,(m_{-i})=\frac{(1+r)y_i+\phi-2\,(1+r)\,E-2(1-\delta)\,(1+r)\,m_{-i}-2\,(1+r)\,\alpha_i\delta\sum_{j\neq i}y_j}{3\,(1+r)} \tag{3}$$

The best response of the other players:

$$\rho_{-i}\,(m_i)=\frac{(1+r)y_{-i}+\phi-2\,(1+r)\,E-2(1-\delta)\,(1+r)\,m_i-2\,(1+r)\,\alpha_{-i}\delta y_i}{3\,(1+r)} \tag{4}$$

Replacing the best response of the others players (4) in equation (3) :

$$\begin{aligned}
& 9\,(1+r)^2\,m_i-4(1-\delta)^2\,(1+r)^2\,m_i \\
= \; & 3(1+r)^2y_i+3\,(1+r)\,\phi-6\,(1+r)^2\,E-2(1-\delta)(1+r)^2y_{-i} \\
& -2(1-\delta)\,(1+r)\,\phi+4(1-\delta)\,(1+r)^2\,E \\
& +4(1-\delta)\,(1+r)^2\,\alpha_{-i}\delta y_i-6\,(1+r)^2\,\alpha_i\delta\sum_{j\neq i}y_j
\end{aligned}$$

$$\begin{aligned}
& [3-2\,(1-\delta)]\,[3+2\,(1-\delta)]\,(1+r)^2\,m_i \\
= \; & 3(1+r)^2y_i+3\,(1+r)\,\phi-6\,(1+r)^2\,E-2(1-\delta)(1+r)^2y_{-i}-2(1-\delta)\,(1+r)\,\phi+4(1-\delta)\,(1+r)^2\,E \\
& +4(1-\delta)\,(1+r)^2\,\alpha_{-i}\delta y_i-6\,(1+r)^2\,\alpha_i\delta\sum_{j\neq i}y_j
\end{aligned}$$

$$m_i^*=\frac{(3+4\alpha_{-i}\delta(1-\delta))\,(1+r)y_i+(1-2\delta)\,\phi-2\,(1+r)\,(1+2\delta)\,E-2(1+r)\sum_{j\neq i}y_j\,[(1-\delta)+3\alpha_i\delta]}{(1+r)\,[3-2\,(1-\delta)]\,[3+2\,(1-\delta)]}$$

Proof of Proposition 4.4.1 :

Best response of the optimist is given by:

$$\rho_o\,(m_p)=\frac{(1-\delta)y-\psi(E)-(1-\delta)m_p}{2} \tag{5}$$

Best response of the pessimist is given by:

$$\rho_p\,(m_o)=\frac{y-\psi(E)-(1-\delta)m_o-\delta m_s}{2} \tag{6}$$

Replacing equation (5) in equation (6)

$$4m_p=2y-2\psi(E)-(1-\delta)\,[(1-\delta)\,y-\psi(E)-(1-\delta)m_p]-2\delta m_s$$

$$m_p^{s*} = \frac{\left(1+2\delta-\delta^2\right)y-(1+\delta)\,\psi(E)-2\delta m_s}{(3-\delta)(1+\delta)}$$

$$\begin{aligned}
2m_o &= (1-\delta)\,y-\psi(E)-(1-\delta)\left[\frac{y-\psi(E)-(1-\delta)m_o-\delta m_s}{2}\right] \\
m_o &= \frac{2\,(1-\delta)\,y-(1-\delta)y-2\psi(E)+(1-\delta)\psi(E)+\delta(1-\delta)m_s}{4-(1-\delta)^2}
\end{aligned}$$

$$m_o^{s*} = \frac{(1-\delta)\,y-(1+\delta)\,\psi(E)+\delta(1-\delta)m_s}{(3-\delta)(1+\delta)}$$

Deriving conditions for which $m_o^*\,(m_s) \geqslant m_s$, and $m_p^*\,(m_s) \geqslant m_s$

$$m_o^*\,(m_s) \geqslant m_s$$

$$\frac{(1-\delta)\,y-(1+\delta)\,\psi(E)+\delta(1-\delta)m_s}{(3-\delta)(1+\delta)} \geqslant m_s$$

$$\begin{aligned}
(1-\delta)\,y-(1+\delta)\,\psi(E)+\delta(1-\delta)m_s &\geqslant (3-\delta)(1+\delta)m_s \\
(1-\delta)\,y-(1+\delta)\,\psi(E) &\geqslant (3-\delta)(1+\delta)m_s-\delta(1-\delta)m_s
\end{aligned}$$

$$m_s\,(m_o^* \geqslant m_s) \leq \frac{(1-\delta)\,y-(1+\delta)\,\psi(E)}{3+\delta}$$

$$m_p^*\,(m_s) \geqslant m_s$$

$$\begin{aligned}
\frac{\left(1+2\delta-\delta^2\right)y-(1+\delta)\,\psi(E)-2\delta m_s}{(3-\delta)(1+\delta)} &\geq m_s \\
\left(1+2\delta-\delta^2\right)y-(1+\delta)\,\psi(E)-2\delta m_s &\geq (3-\delta)(1+\delta)m_s \\
\left(1+2\delta-\delta^2\right)y-(1+\delta)\,\psi(E) &\geq (3-\delta)(1+\delta)m_s+2\delta m_s
\end{aligned}$$

$$m_s\left(m_p^* \geqslant m_s\right) \leq \frac{\left(1+2\delta-\delta^2\right)y-(1+\delta)\,\psi(E)}{3+4\delta-\delta^2}$$

Comparing $m_s\left(m_p^* \geqslant m_s\right)$ and $m_s\,(m_o^* \geqslant m_s)$

$$
\begin{aligned}
& m_s\left(m_p^* \geqslant m_s\right) - m_s\left(m_o^* \geqslant m_s\right) \\
= & \frac{\left(1+2\delta-\delta^2\right) y-(1+\delta)\,\psi(E)}{3+4\delta-\delta^2} - \frac{(1-\delta)\,y-(1+\delta)\,\psi(E)}{3+\delta} \\
= & \frac{2\left(3+2\delta-\delta^2\right)\delta y+\left(3+2\delta-\delta^2\right)\delta\psi(E)}{\left(3+4\delta-\delta^2\right)(3+\delta)} \geqslant 0
\end{aligned}
$$

For a $\delta \in [0,1]$, the difference between is always positive and therefore $m_s\left(m_p^* \geqslant m_s\right) \geqslant m_s\left(m_o^* \geqslant m_s\right)$

When $m_s \geqslant \frac{(1-\delta)y-(1+\delta)\psi(E)}{3+\delta}$, the optimist is constrained to contribute m_s. The pessimist will play a best-response to the strategy of his opponent given by $\rho_p(m_s) = \frac{y-\psi(E)-m_s}{2}$.

Condition for which $\rho_p(m_s) \geqslant m_s$

$$
\rho_p(m_s) \geqslant m_s
$$

$$
\begin{aligned}
\frac{y-\psi(E)-m_s}{2} & \geqslant m_s \\
y-\psi(E) & \geqslant 3m_s
\end{aligned}
$$

$$
m_s \leqslant \frac{y-\psi(E)}{3}
$$

Proof of Proposition 10:

The Choquet utility function to be maximized is given by:

$$
V_i(m_i, m_{-i}) = \frac{(1+r)}{(1+\tau)}(y_i+t_i-m_i)\left[E+m_i+(1-\delta)m_{-i}+\alpha_i\delta\sum_{j\neq i}(y_j+t_j)\right]
$$

f.o.c.

$$
\begin{aligned}
y_i+t_i-E-2m_i-(1-\delta)m_{-i}-\alpha_i\delta\sum_{j\neq i}(y_j+t_j) &= 0 \\
y_i+t_i-E-(1-\delta+1+\delta)\,m_i-(1-\delta)m_{-i}-\alpha_i\delta\sum_{j\neq i}(y_j+t_j) &= 0 \\
y_i+t_i-E-(1+\delta)\,m_i-(1-\delta)M-\alpha_i\delta\sum_{j\neq i}(y_j+t_j) &= 0 \\
\sum_{i=1}^{n}\left[y_i+t_i-E-(1+\delta)\,m_i-(1-\delta)M-\alpha_i\delta\sum_{j\neq i}(y_j+t_j)\right] &= 0 \\
y+T-nE-(1+\delta)\,M-n(1-\delta)M-\delta\sum_{i=1}^{n}\alpha_i\sum_{j\neq i}(y_j+t_j) &= 0
\end{aligned}
$$

Here we will assume that $t_i = t$, for all i, the lump sum transfers do not depend on income or on optimism/pessimism.

$$y + T - nE - (n+1+\delta(n-1))\,M - \sum_{i=1}^{n}\alpha_i\delta\left[\sum_{j\neq i} y_j + (n-1)t\right] = 0$$

$$y - nE - (n+1+\delta(n-1))\,M - \sum_{i=1}^{n}\alpha_i\delta\sum_{j\neq i} y_j + \left[1 - \sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]T = 0$$

Next, $T = \tau(y - M)$

$$y - nE - (n+1+\delta(n-1))\,M - \sum_{i=1}^{n}\alpha_i\delta\sum_{j\neq i} y_j + \left[1 - \sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\tau(y-M) = 0$$

$$M(\tau) = \frac{\left[1 + \left[1 - \sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\tau\right]y - nE - \sum_{i=1}^{n}\alpha_i\delta\sum_{j\neq i} y_j}{(n+1+\delta(n-1)) + \left[1 - \sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]}$$

$$\frac{\partial M}{\partial \tau} = \frac{\left[1 - \sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]y}{(n+1+\delta(n-1)) + \left[1 - \sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]} > 0$$

$$C = (1+r)(y - M)$$

$$C = (1+r)\left[\frac{\left[(n+1+\delta(n-1)) - \tau - \sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}(1-\tau)\right]y + nE + \sum_{i=1}^{n}\alpha_i\delta\sum_{j\neq i} y_j}{(n+1+\delta(n-1)) + \left[1 - \sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]}\right]$$

$$\frac{\partial C}{\partial \tau} = (1+r)\frac{-1 + \sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}}{\left[(n+1+\delta(n-1)) + \left[1 - \sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\right]} < 0$$

Proof of proposition 11:

$$V^S = (1+r)\sum_{i=1}^{n}(y_i - m_i)\left[E + m_i + (1-\delta)\sum_{j\neq i} m_j + \alpha_i\delta\sum_{j\neq i} y_j\right]$$

$$\frac{\partial V^S}{\partial m_i} = y_i - 2m_i - E - (1-\delta)\sum_{j\neq i} m_j - \alpha_i\delta\sum_{j\neq i} y_j + (1-\delta)\sum_{j\neq i}(y_j - m_j) = 0$$

$$y_i - 2\delta m_i - E - 2(1-\delta)\,M + (1-\delta-\alpha_i\delta)\sum_{j\neq i} y_j = 0$$

$$y - 2\delta M - nE - 2n(1-\delta)\,M + n(1-\delta)\sum_{j\neq i} y_j - \delta\sum_{i=1}\alpha_i\sum_{j\neq i} y_j$$

$$y+n\,(1-\delta)\sum_{j\neq i}y_j-nE-\delta\sum_{i=1}\alpha_i\sum_{j\neq i}y_j=2\left[\delta+n(1-\delta)\right]M^S$$

$$M^S=\frac{\left[1+(1-\delta)\,(n-1)\right]y-nE-\delta\sum_{i=1}^{n}\alpha_i\sum_{j\neq i}y_j}{2\left[\delta+n(1-\delta)\right]}$$

Proof of proposition 12:

To find the Pareto-optimal tax rate, we have to equalize Pareto-optimal mitigation to equilibrium mitigation with taxes.

$$\begin{aligned}
M^{SO} &= M(\tau^{SO}) \\
& \frac{\left[1+(1-\delta)\,(n-1)\right]y-nE-\delta\sum_{i=1}^{n}\alpha_i\sum_{j\neq i}y_j}{2\left[\delta+n(1-\delta)\right]} \\
&= \frac{\left[1+\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right]\tau^{SO}\right]y-nE-\delta\sum_{i=1}^{n}\alpha_i\sum_{j\neq i}y_j}{(n+1+\delta(n-1))+\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right]}
\end{aligned}$$

$$\begin{aligned}
& (n+1+\delta(n-1))\left[1+(1-\delta)\,(n-1)\right]y+\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\left[1+(1-\delta)\,(n-1)\right]y \\
& -n\left[(n+1+\delta(n-1))+\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\right]E \\
& -\delta\left[(n+1+\delta(n-1))+\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\right]\sum_{i=1}^{n}\alpha_i\sum_{j\neq i}y_j \\
= \;& 2\left[\delta+n(1-\delta)\right]\left[1+\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\tau^{SO}\right]y \\
& -2\left[\delta+n(1-\delta)\right]n\psi(E)-2\left[\delta+n(1-\delta)\right]\sum_{i=1}^{n}\alpha_i\delta\sum_{j\neq i}y_j
\end{aligned}$$

$$\begin{aligned}
& (n+1+\delta(n-1))\left[1+(1-\delta)\,(n-1)\right]y \\
& +\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\left[1+(1-\delta)\,(n-1)\right]y-2\left[\delta+n(1-\delta)\right]\left[1+\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\tau^{SO}\right]y \\
= \;& n\left[(n+1+\delta(n-1))+\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]-2\left[\delta+n(1-\delta)\right]\right]E \\
& +\delta\left[(n-1)(3\delta-1)+\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\right]\sum_{i=1}^{n}\alpha_i\sum_{j\neq i}y_j
\end{aligned}$$

$$
\begin{aligned}
& (n+1+\delta(n-1))\,[n-\delta(n-1)]\,y \\
& +\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right][n-\delta(n-1)]\,y \\
& -2\,[n-\delta\,(n-1)]\left[1+\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\tau^{SO}\right]y \\
= \; & n\left[(1-n)\,(1-3\delta)+\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\right]E \\
& +\delta\left[(n-1)(3\delta-1)+\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\right]\sum_{i=1}^{n}\alpha_i\sum_{j\neq i}y_j
\end{aligned}
$$

$$
\begin{aligned}
& \left[(n-1)\,(1+\delta)+\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\right][n-\delta(n-1)]\,y \\
& -2\,[n-\delta\,(n-1)]\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]\tau^{SO}y \\
= \; & \left[\left[1-\sum_{i=1}^{n}\alpha_i\delta\frac{(n-1)}{n}\right]-(n-1)\,(1-3\delta)\right]\left[nE+\delta\sum_{i=1}^{n}\alpha_i\sum_{j\neq i}y_j\right]
\end{aligned}
$$

$$
\tau^{SO}=\frac{\left[\frac{(n-1)(1+\delta)}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right]}+1\right]y-\left[1-\frac{(n-1)(1-3\delta)}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right][n-\delta(n-1)]}\right]\left[nE+\delta\sum_{i=1}^{n}\alpha_i\sum_{j\neq i}y_j\right]}{2y}
$$

Let 's now verify the sign of $\frac{\partial\tau^{S}}{\partial\sum_{i=1}^{n}\alpha_i}$

For sake of simplicity let's assume that:

$y_i = const \;\; = \frac{y}{n}$

$$
\tau^{SO}=\frac{\left[\frac{(n-1)(1+\delta)}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right]}+1\right]y-\left[1-\frac{(n-1)(1-3\delta)}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right][n-\delta(n-1)]}\right]\left[nE+\delta(n-1)y\sum_{i=1}^{n}\alpha_i\right]}{2y}
$$

$$
\begin{aligned}
\tau^{SO} \; = \; & \frac{\left[\frac{(n-1)(1+\delta)}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right]}+1-\delta n\sum_{i=1}^{n}\alpha_i+\delta\sum_{i=1}^{n}\alpha_i+\frac{(n-1)(1-3\delta)\delta(n-1)\sum_{i=1}^{n}\alpha_i}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right][n-\delta(n-1)]}\right]y}{2y} \\
& +\frac{\left[\frac{(n-1)(1-3\delta)}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right][n-\delta(n-1)]}-1\right]nE}{2y}
\end{aligned}
$$

$$\tau^{SO} = \frac{\left[\frac{(n-1)(1+\delta)}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right]}+1\right]y+\left[-1+\frac{(n-1)(1-3\delta)}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right][n-\delta(n-1)]}\right]nE}{2y}$$

$$+\frac{y\delta\left(n-1\right)\sum_{i=1}^{n}\alpha_i\left[-1+\frac{(n-1)(1-3\delta)}{\left[1-\delta\frac{(n-1)}{n}\sum_{i=1}^{n}\alpha_i\right][n-\delta(n-1)]}\right]}{2y}$$

set $x = \sum_{i=1}^{n}\alpha_i$

$$\tau^{SO} = \frac{\left[\frac{(n-1)(1+\delta)}{\left[1-\delta\frac{(n-1)}{n}x\right]}+1\right]y+\left[-1+\frac{(n-1)(1-3\delta)}{\left[1-\delta\frac{(n-1)}{n}x\right][n-\delta(n-1)]}\right]nE+y\delta\left(n-1\right)x\left[-1+\frac{(n-1)(1-3\delta)}{\left[1-\delta\frac{(n-1)}{n}x\right][n-\delta(n-1)]}\right]}{2y}$$

$$\tau^{SO} = \frac{\frac{(n-1)(1+\delta)}{\left[1-\delta\frac{(n-1)}{n}x\right]}y+\frac{(n-1)(1-3\delta)}{\left[1-\delta\frac{(n-1)}{n}x\right][n-\delta(n-1)]}nE+y\delta\left(n-1\right)x\left[-1+\frac{(n-1)(1-3\delta)}{\left[1-\delta\frac{(n-1)}{n}x\right][n-\delta(n-1)]}\right]}{2y}+\frac{y-nE}{2y}$$

$$\frac{\partial\tau^{SO}}{\partial x} = \frac{\delta\frac{(n-1)}{n}}{\left[1-\delta\frac{(n-1)}{n}x\right]^2}\left[\left(n-1\right)\left(1+\delta\right)y+\frac{\left(n-1\right)\left(1-3\delta\right)}{[n-\delta(n-1)]}nE\right]$$

$$+y\delta\left(n-1\right)\left[-1+\frac{\left(n-1\right)\left(1-3\delta\right)}{\left[1-\delta(n-1)\frac{x}{n}\right]^2[n-\delta(n-1)]}\right]>0$$

$$\frac{1}{n}\left[\left(n-1\right)\left(1+\delta\right)y+\frac{\left(n-1\right)\left(1-3\delta\right)}{[n-\delta(n-1)]}nE\right]+y\left[-\left[1-\delta(n-1)\frac{x}{n}\right]^2+\frac{\left(n-1\right)\left(1-3\delta\right)}{[n-\delta(n-1)]}\right]>0$$

$$\frac{1}{n}\left[\left(n-1\right)\left(1+\delta\right)y+\frac{\left(n-1\right)\left(1-3\delta\right)}{[n-\delta(n-1)]}nE\right]+y\left[-\left[1-\delta(n-1)\frac{x}{n}\right]^2+\frac{\left(n-1\right)\left(1-3\delta\right)}{[n-\delta(n-1)]}\right]>0$$

$$\frac{1}{n}\left[\left(n-1\right)\left(1+\delta\right)y+\frac{\left(n-1\right)\left(1-3\delta\right)}{[n-\delta(n-1)]}nE\right]+y\frac{\left(n-1\right)\left(1-3\delta\right)}{[n-\delta(n-1)]}>y\left[1-\delta(n-1)\frac{x}{n}\right]^2$$

$$\left|1-\delta(n-1)\frac{x}{n}\right|<\sqrt{\frac{1}{n}\left[\left(n-1\right)\left(1+\delta\right)+\frac{\left(n-1\right)\left(1-3\delta\right)}{[n-\delta(n-1)]}\frac{n\psi\left(E\right)}{y}\right]+\frac{\left(n-1\right)\left(1-3\delta\right)}{[n-\delta(n-1)]}}$$

Since

$$1-\delta(n-1)\frac{x}{n} = n\left(1-\delta x\right)+\delta x>0$$

we have that

$$\left|1-\delta(n-1)\frac{x}{n}\right| = 1-\delta(n-1)\frac{x}{n}$$

Hence, $\frac{\partial \tau^{SO}}{\partial x} > 0$ will hold if "average optimism", $\frac{x}{n} = \frac{\sum_{i=1}^{n} \alpha_i}{n}$ is sufficiently large:

$$\frac{1}{\delta(n-1)} - \frac{\sqrt{\frac{1}{n}\left[(n-1)(1+\delta) + \frac{(n-1)(1-3\delta)}{[n-\delta(n-1)]}\frac{nE}{y}\right] + \frac{(n-1)(1-3\delta)}{[n-\delta(n-1)]}}}{\delta(n-1)} < \frac{x}{n}$$

In particular, if the l.h.s. is negative, this will be the case for all $x \in [0,1]$.

Note that

$$\left(-\left[1-\delta(n-1)\frac{x}{n}\right]^2\right)'_x = \frac{2\delta(n-1)}{n}\left[1-\delta(n-1)\frac{x}{n}\right] > 0$$

is increasing in x. If $x = 0$, this term has a minimum, which is:

$$\frac{1}{n}\left[(n-1)(1+\delta)y + \frac{(n-1)(1-3\delta)}{[n-\delta(n-1)]}nE\right] + y\left[-1 + \frac{(n-1)(1-3\delta)}{[n-\delta(n-1)]}\right] > 0$$

$$\left[\frac{1}{n}(n-1)(1+\delta) - 1\right]y + \frac{1}{n}\frac{(n-1)(1-3\delta)}{[n-\delta(n-1)]}nE + y\frac{(n-1)(1-3\delta)}{[n-\delta(n-1)]} > 0$$

$$\begin{aligned}\left[\frac{1}{n}(n-1)(1+\delta) - 1\right] &> 0 \\ (n-1)(1+\delta) - n &> 0\end{aligned}$$

$$1 + \delta > \frac{n}{n-1}$$

But if $n \to \infty$, $\frac{n}{n-1} \to 1$ and hence, $1+\delta > \frac{n}{n-1}$ for large n's. $\frac{4}{3} > \frac{n}{n-1}$, implies that for $n > 4$, the entire term is positive. Hence, for more than 4 players, and any $\delta \in \left(0, \frac{1}{3}\right)$ such that $1 + \delta > \frac{n}{n-1}$, the tax depends positively on the degree of optimism.

if $\delta < \frac{1}{3}$ and $n \leq 4$, the effect of optimism on the tax will be negative.