Coordination of abatement and policy across interconnected sectors

Preliminary Draft

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Abstract

To drastically reduce GHG emission, numerous specific measures are required in all sectors of the economy. These measures, and the GHG consequences of their implementations, are not independent one from the other because of sectoral linkages. For instance, the carbon footprint of electric vehicles depends on the electricity mix, an issue that have received considerable attention but few economic analysis. The present paper address the issue of sectoral policy coordination, especially with pigovian carbon pricing is unavailable.

It analyzes the optimal allocation of mitigation effort among two vertically connected sectors, an upstream (e.g. electricity) and a downstream (e.g. transportation) one. The clean downstream technology (e.g. electric vehicle) consumes the upstream production and may shift production to that sector. Using a simple partial equilibrium model, we connects the concept of Marginal Abatement Cost (MAC) and Life-Cycle-Assessment. We propose a characterization that indicates the order of options implementations, which is relevant for policy making. The decentralized version of the model allows us to characterize optimal second-best policy in presence of imperfect GHG taxation. We find conditions of policy coordination for various price and quantity instruments settings.

1 Introduction

The reduction of Greenhouse Gas (GHG) emissions requires to shift from fossil energy to non-carbon energy. For many energy uses (e.g. transport, industry, heating) such a shift is done via the use of electricity as a vector of non-carbon energy (e.g. nuclear, renewable), and mitigation policies need to coordinate the decarbonization of the upstream electricity sector with the electrification of the downstream sector. Indeed, as long as the upstream sector is not fully decarbonized, the decarbonization of downstream activities partly shift GHG emissions upstream, Life Cycle Assessments (LCA) of electric vehicles have stressed such effect (e.g. Archsmith et al., 2015) raising concerns about their carbon footprint. The purpose of the present article is twofold. First it aims to analyze the optimal allocation and sequencing of mitigation efforts between two polluting sectors when the pollution abatement of one, downstream sector, is done by consuming the production of the other, upstream sector. Second, it investigates whether such linkages creates a need of policy coordination across sectors, especially when pigovian tax are unavailable.

We develop a partial equilibrium model with two sectors: an upstream and a downstream one. In each sector, a dirty and a clean technology are available with the clean downstream technology (e.g. electric cars) consuming, as an input, the upstream good (e.g. electricity). We analyze the optimal allocation of production for a given Social Cost of Carbon (SCC), and the optimal sequencing of the deployment of the two clean technologies as the SCC increases. We build a MAC-curve that incorporates the linkage between the two sectors, and consider policy consequences, notably the optimal second-best subsidy of the downstream clean technology when GHG emissions are imperfectly priced in the economy.

The connection of the two sectors has two consequences on the optimal allocation. First the upstream sector produces more than with no SCC, its production is clean or dirty. Second, the marginal cost of the downstream technology is endogenously determined and depends on the SCC. We establish a condition (Proposition 1) under which upstream emissions increases with the SCC because of the demand emanating from the downstream sector. Concerning MAC-curves (SCC as a function of abatement from 0 to total initial emissions), the two consequences mentioned above translate into: an increased "potential" of the upstream sector, total clean production is larger than the initial size of the sector. And an adjusted MAC in the downstream sector that incorporates the marginal cost and emission intensity of the upstream sector. This last point contribute to bridge the gap between economics and LCA approaches.

Regarding the sequencing of decarbonization as the SCC increases, it is relatively easy to determine which sector starts and finishes first based on cost functions and sector sizes (Proposition 2). However, the sequencing along the MAC-curve is more subtle and we exhaustively identify conditions for a each possible sequencing (Propositions 3). Notably, we exhibit a "transient decarbonisation" pathway along which, as the SCC increases, the upstream sector is fully decarbonized before recarbonizing and eventually being fully clean again.

Finally, we investigate the policy consequences of inter-sectoral linkages, in a flexible realistic policy environment. Indeed, a comprehensive carbon pricing enforces the first best allocation, and LCA are not needed. Without carbon pricing and only subsidy to clean technologies, both downstream and upstream, the first best cannot be reached except with inelastic demand. The optimal downstream subsidy depends on the marginal upstream technology. If the marginal upstream technology is dirty, and whatever its market share, the optimal downstream subsidy is decreasing with respect to the upstream GHG emissions and LCA is relevant. If the marginal upstream technology is clean, the optimal downstream subsidy is decreasing with respect to the upstream subsidy.

The present article is related to several strand of the literature and we organize our review from the applied work (LCA), sometime grey literature, to the more conceptual consideration (second best). Before proceeding, we would like to stress that one of our contribution is precisely to bridge gaps between the applied work routinely used to device, and debate about, climate policies and economic theory.

Life Cycle Assessments regularly temper the enthusiasm associated with new "green" technology because of their upstream footprint. Our primary motivation comes from the deployment of electric vehicles and the debates surrounding their total environmental benefits. Several authors have analyzed the upstream carbon footprint associated with electricity production and the battery production (Archsmith et al., 2015). We only consider one upstream sector and a two sector situation but the analysis could be developed along an input-output framework. There are other examples: hydrogen is another vector of energy mostly produced with a carbon emitting technology, and has application to decarbonize downstream energy uses (transport, heating); a more prospective example is artificial meat, the production of which require a lot of energy but would abate cattle emissions (Tuomisto and Teixeira de Mattos, 2011; Mattick et al., 2015).

MAC curves rank sectoral mitigation options by their abatement potentials and by their abatement costs. A logical recommendation that follows is to schedule investments from the cheapest. MAC curves have faced several criticism by scholar (Kesicki and Strachan, 2011; Kesicki and Ekins, 2012). They fail at capturing relevant dynamic effects such as sectoral inertia, technological learning and, in our case, sectoral interactions. If several authors tackled the first two issues (Vogt-Schilb and Hallegatte, 2014; Vogt-Schilb et al., 2018; Creti et al., 2018), the third has not been studied to our knowledge. Note also that MAC curves usually fail to take into account demand elasticity and are focused on technology substitution.

Macro modeling: IAM modeling (Acemoglu et al., 2012; Golosov et al., 2014; Gerlagh and Liski, 2018) consider an energy sector that combine carbon and non-carbon energy sources, technology chains are not modeled and the coordination issue studied is not an issue

Second Best policies Seminal work of Lipsey and Lancaster (1956) establish how in a second best world, with pre-existing distortion in the economy, optimal formulas should be modified. More recently and applied to the transportation sector Holland et al. (2015) analyze the optimal subsidy of electric vehicle when electricity production is unregulated,[1], they stress that optimal subsidy of electric vehicle should integrate the external costs associated to energy production. Their contribution is not theoretical. They do not consider the impact of electricity regulation, and possibly the joint optimization of subsidies. The analysis of second-best policies with imperfect pricing of externality is well developed in the literature on waste management and recycling (eg Walls and Palmer, 2001)

The article is organized as follow. The model is introduced in Section 1. Then, in Section

[1] They do not discuss regulation in the electricity sector and only consider NOx SOx permit market in an appendix...

2 we analyze the optimal allocation and MAC-curves. Policy considerations are described in Section 3. Section 4 discusses some limitations and possible extension. Section 5 concludes.

2 The analytical framework

2.1 The model

We consider a partial equilibrium model with two interrelated sectors an upstream (e.g. electricity) and a downstream (e.g. mobility) sector. There are two goods, $i = m, e$ (m stands for mobility, and e for electricity), both consumed by households. For both goods there is a dirty and a clean technology, and the clean downstream technology uses the upstream good (electricity is both consumed by households and by electric cars).

For each sector $i = m, e$ the total quantity consumed by households is Q_i, the associated gross consumers surplus is $S_i(Q_i)$, with $S_i' > 0$, and $S_i'' < 0$.

On the production side: in sector $i = m, e$ the total quantity produced is $q_{id} + q_{ic}$ the sum of dirty and clean productions, with production costs $C_{ij}(q_{ij})$, with $i = m, e$ and $j = d, c$. Cost functions are increasing and convex, $C_{ij}' > 0$ and $C_{ij}'' \geq 0$.[2] Each clean downstream unit consume θ units of the upstream good so that the total quantity produced $q_{ed} + q_{ec}$ is equal to the quantity consumed by households Q_e and by the downstream clean variety θq_{mc}: $q_{ed} + q_{ec} = Q_e + \theta q_{mc}$.

The production of a dirty unit in sector i emits α_i tons of CO_2, and we denote μ (in \$ per tCO_2) the "Social Cost of Carbon". Total welfare is then

$$W(\mathbf{q}, \mu) = \sum_i S_i(Q_i) - \sum_{ij} C_{ij}(q_{ij}) - \mu[\alpha_m q_{md} + \alpha_d q_{ed}] \tag{1}$$

subject to $Q_m = q_{md} + q_{mc}$ and $Q_e + \theta q_{mc} = q_{ed} + q_{ec}$.

Our objective is to understand the allocation of efforts between the upstream and downstream sector as a function of the SCC μ, and build a MAC-curve that integrates the interaction between the clean downstream variety and the upstream sector. Indeed, the MAC-curve

[2] Justification of convex costs: increasing storage costs and decreasing efficiency (eg wind) of renewabl electricity, transport increasingly costly as the scope (light vehicle less expensive to decarbonize than vans, trucks, ships...)

is the inverse of the CO_2 demand function. We have in mind a dynamic context of an energetic transition in which the SSC progressively increases over time (for instance due to a carbon budget consistent with the Paris agreement), and the economy is progressively decarbonized. At the end of the transition the whole economy is clean and $q_{id} = 0$ in both $i = m$ and e. Along the transition the MAC-curve indicates when to do what mitigation effort, and we are particularly interested by the ranking of the respective starting and ending dates of the two sectors.

Let us first introduce the following assumptions. The first ensures that without environmental cost ($\mu = 0$) there is no clean production, and the second that there is nonnegative clean production for large SCC.

Assumption 1 *There are $Q_m^0 > 0$ and $Q_e^0 > 0$ such that*

$$S_i'(Q_i^0) = C_{id}'(Q_i^0) < C_{ic}'(0) \tag{2}$$

Assumption 2 *There are $Q_m^1 > 0$ and $Q_e^1 > \theta Q_m^1$ such that*

$$S_e'(Q_e^1 - \theta Q_m^1) = C_{ec}'(Q_e^1) \text{ and } S_m'(Q_m^1) = C_{mc}'(Q_m^1) + \theta C_{ec}'(Q_e^1) \tag{3}$$

Several limitations of the model (further discussed in Section **??**): we do not consider the possibility to charge electric cars at night, so that the content of the electricity used to charge is not exactly the same as the total mix of the grid. Notably for hydrogen EV this is promising way of reducing the indirect carbon footprint of electric vehicles. This issue is related to the variability of the demand for electricity and the intermintency of renewable We donot consider imperfect subsitutability between clean and dirty varieties, notably in the downstream sector, in order to focuss on the role of the interconnection. Dynamic issues: adjustment cost, learning-by-doing.

2.2 Quadratic specification

To get explicit formula, some of our results and draw figures, we will make use of the following quadratic specification, in which the cost of dirty technologies is assumed to be linear, gross surplus and clean cost functions quadratic.

Specification 1

$$S_i(Q_i) = a_i Q_i - \frac{1}{2} b_i Q_i^2$$

$$C_{id}(q_{id}) = c_{id} q_{id} \text{ and } C_{ic}(q_{ic}) = c_{ic}^0 q_{ic} + \frac{1}{2\gamma_i} q_{ic}^2$$

with $a_i, b_i, c_{ij}, \gamma_i$ all nonnegative real numbers.

Under this specification, assumptions 1 and 2 respectively corresponds to $c_{id} < c_{ic}$, and to $a_i > C'_{ic}(0)$.

3 Optimal transitions and MAC curves

3.1 Optimum

The social planner maximizes welfare (1), the optimal allocation is a vector $\mathbf{q}^*(\mu) = (q^*_{ij}(\mu))_{i,j}$. Denoting ϕ_{ij} the Lagrange multiplier of the positivity constraint $q_{ij} \geq 0$, the first order conditions are

$$S'_e(q_{ed} + q_{ec} - \theta q_{mc}) = C'_{ed} + \alpha_e \mu \ - \phi_{ed} \tag{4}$$

$$= C'_{ec} - \phi_{ec} \tag{5}$$

$$S'_m(q_{md} + q_{mc}) = C'_{md} + \alpha_m \mu - \phi_{md} \tag{6}$$

$$= C'_{mc} - \phi_{ec} + -\theta S'_e \tag{7}$$

At the optimum allocation in each sector a positive quantity is produced and consumed thanks to Assumption 1 and 2. In each sector marginal consumer surplus is equalizes with the marginal costs of each technology used. The marginal cost of the clean downstream technology encompasses the marginal benefit from the upstream good consumption.

For each sector there are three possible configurations: only dirty ($q^*_{ic} = 0$), only clean ($q_{id} = 0$), both (with $q_{it} > 0$ for $t = d, c$). Overall, there are therefore nine possible configurations summarized in Table 3.1

		Sector m		
		d	b	c
Sector e	d	D	db	dc
	b	bd	B	bc
	c	cd	cb	C

Table 1: Possible Configurations: each box correspond to one configuration the first (second) letter being the upstream (downstream) sector. Each sector can be in three states: only dirty "d", only clean "c", or both "b'.

To better understand the linkage of the two sectors it is useful to introduce sectoral surplus as a function of the downstream price and the SCC. Let us denote $V_m(\mu, P_e)$ and $V_e(\mu, P_e)$ the surplus in the downstream and upstream sector respectively:

$$V_m(\mu, P_e) = \max_{q_{md}\geq 0,\ q_{mc}\geq 0} S_m(q_{md} + q_{mc}) - C_{md}(q_{md}) - C_{mc}(q_{mc}) - P_e\theta q_{mc} - \mu q_{md} \quad (8)$$

$$V_e(\mu, P_e) = \max_{\substack{q_{ed}\geq 0,\ q_{ec}\geq 0 \\ Q_e\leq q_{ed}+q_{ec}}} P_e.(q_{ed} + q_{ec} - Q_e) + S_e(Q_e) - C_{ed}(q_{ed}) - C_{ec}(q_{ec}) - \mu q_{ed} \quad (9)$$

The optimal allocation can be determined in two steps, first maximizes the sectoral surpluses for a given price P_e and then find the price that equalizes the upstream demand with the downstream extra production: $\theta q_{mc} = q_{ed} + q_{ec} - Q_e$. The second step equalizes the marginal value of the upstream good in the two sectors $\theta S'_e = S'_m - C'_{mc}$, and could be interpreted as a market clearing condition. The marginal surplus from electricity at the optimum $\mathbf{q}^*$ is $P_e^*(\mu)$:

$$P_e^* \equiv S'_e(q_{ed}^* + q_{ec}^* - \theta q_{mc}^*)$$

and $W(\mathbf{q}^*, \mu) = \sum_i V_i(\mu, P_e^*(\mu))$.

Lemma 1 *If $\theta\alpha_e > \alpha_m$, there is no clean downstream production if there is some dirty upstream production: $q_{ed}^*(\mu) > 0 \Rightarrow q_{mc}^*(\mu) = 0$.*

Proof:

Lemma 2 *As μ increases:*

- *The marginal value of the upstream good $P_e^*(\mu)$ increases;*
- *Clean production increases in the upstream sectors;*
- *Dirty production decreases in the downstream sector.*

Because of the increased demand of the upstream good coming from the downstream sector, emissions from the upstream sector can increase with respect to the SCC. It can indeed happen when the upstream sector is fully dirty (configurations db and dc) but even when the clean upstream technology is used.

Proposition 1 *As the social cost of carbon μ increases the quantity of emissions in the upstream sector increases in configuration B if and only if:*

$$\frac{\theta\big(\alpha_m - \theta\alpha_e(1+\epsilon)\big)\big)}{C''_{md} + (1+\epsilon)C''_{mc}} > \alpha_e \left(\frac{1}{-S''_e} + \frac{1}{C''_{ec}}\right) \tag{10}$$

in which $\epsilon = \frac{C''_{md}}{-S''_m}$

Proof in Appendix A

3.2 Pathways and MACs

We name *transition pathway*, the succession of states when carbon price increase from zero until full decarbonization of all sectors. In this section, we determine how the possible pathways are related to the ordering of sectoral MACs. Let us define two thresholds of social cost of carbon SCC for each sectors: the one at which the transition starts and the one at which it ends.

Definition 1 *For each sector $i = m, e$ let us define the two SCC $\underline{\mu}_i$ and $\bar{\mu}_i$ such that*

$$\underline{\mu}_i = \max\{\mu \mid q^*_{ic}(\mu) = 0\} \text{ and } \bar{\mu}_i = \sup\{\mu \mid q^*_{id}(\mu) > 0\} \tag{11}$$

The ordered sequence of these thresholds defines the possible transition pathways. For instance, the sequence $\underline{\mu}_m < \bar{\mu}_m < \underline{\mu}_e < \bar{\mu}_e$ corresponds to the pathway $D- > db- > dc- > bc- > C$ with the notations of Table 3.1, and means that the downstream sector is fully decarbonized before the upstream start its own transition. By definition these thresholds are unique. However in principle, nothing prevent transitions pathways to be composed of more than six states. Note also that size of the upstream sector may increase for $\mu > \bar{\mu}_e$ if the downstream sector has not been decarbonized for those values of carbon price.

In what follows, the results will be based on specification 1. We introduce additional meaningful notations with:

$$Q_i^0 = \frac{a_i - c_{id}}{b_i}$$
$$M_e(Q_e) = \frac{C'_{ec}(Q_e) - c_{ed}}{\alpha_e}$$
$$M_m(Q_m) = \frac{C'_{mc}(Q_m) + \theta c_{ed} - c_{md}}{\alpha_m - \theta\alpha_e}$$
$$x = \frac{\alpha_m}{\alpha_e}$$

Q_i^0 refers to the sector size without any carbon price. x refers the ratio of sector emissions rates. We name functions M_e and M_m *abatement costs functions.* of the upstream and downstream sectors. Note that the abatement cost function includes interactions effects with an effective emission rate $\alpha_m - \theta\alpha_e$ and the full clean technology cost $C'_{mc}(QM) + \theta c_{ed}$.

Furthermore, we introduce some last notations relating to the interplay between cost convexities and demand elasticities:

$$f_i = \frac{1}{\gamma_i b_i} \; g = \frac{\theta}{\gamma_e b_m}$$

.

Proposition 2 *Transition pathways follows:*

(i) upstream starts before downstream starts ($\underline{\mu}_e < \underline{\mu}_m$) *iff* $M_e(0) < M_m(0)$

(ii) upstream ends after downstream ends ($\bar{\mu}_e < \bar{\mu}_m$) *iff*

$\frac{1}{1+f_e+gx} M_e(Q_e^0 + \theta Q_m^0) < \frac{x-\theta}{(1+f_m)x-\theta} M_m(Q_m^0)$

This proposition related the part of the pathways properties with a metric that we can interpret as effective abatement costs. As stated before, M_e and M_m are abatement costs function. The values of the different thresholds are specific to each pathway. These are listed in the appendix. When demands are inelastic, f_i and g go to the zero and hence comparisons of threshold are directly related to the abatement cost functions. With elastic demands, the sizes of each sector decrease with the carbon price, until at least full decarbonization. In fact when demand are elastic, lowering demand is an additional option to decarbonize sectors. There is then an interplay between the demand elasticity and the convexity of clean technologies, represented by $f_i = \frac{1}{b_e \gamma_i}$. This interplay is reflected in the proposition by a decreasing factor in front of abatement cost functions. Furthermore, the decreasing factor of the abatement functions of the upstream sector may include $g = \frac{\theta}{b_m \gamma_e}$ which reflects the interplay between the convexity of upstream cost and the elasticity of the downstream demand.

The comparison of $\bar{\mu}_i$ and $\underline{\mu}_j$ is more subtle, and three regimes appears with distinct properties:

Proposition 3 *Transition pathways are determined by the following relations:*

One-way *When $\theta > \frac{\alpha_m}{\alpha_e} = x$, there is a single pathway is possible in which the upstream sector is fully decarbonized before the beginning of the downstream transition.*

Regular *When $\theta(x - \theta)\frac{\gamma_m}{\gamma_e} < 1 + f_e$ there are six possible pathways, characterized by:*

- *(i) upstream ends before downstream starts ($\bar{\mu}_e < \underline{\mu}_m$) iff $\frac{1}{1+f_e} M_e(Q_e^0) < M_m(0)$*
- *(ii) downstream ends before upstream starts ($\bar{\mu}_m < \underline{\mu}_e$) iff $\frac{x-\theta}{(1+f_m)x-\theta} M_m(Q_m^0) < M_e(0)$*

Recarb *When $\theta(x - \theta)\frac{\gamma_m}{\gamma_e} > 1 + f_e$ and $x > \theta$, there a five pathways characterized by:*

- *(i) upstream can not end during the transition of downstream*
- *(ii) upstream ends before downstream starts ($\bar{\mu}_e < \underline{\mu}_m$) iff*
  $M_e(0) < M_m(0)$ and $\frac{1}{1+f_e+gx} M_e(Q_e^0 + \theta Q_m^0) < \frac{x-\theta}{(1+f_m)x-\theta} M_m(Q_m^0)$
- *(iii) downstream ends before upstream starts ($\bar{\mu}_m < \underline{\mu}_e$) iff $\frac{x-\theta}{(1+f_m)x-\theta} M_m(Q_m^0) < M_e(0)$*

(iv) *when the downstream start and ends during the transition of upstream, there may be* <u>*transient upstream decarbonization*</u> *iff* $\frac{1}{1+f_e} M_e(Q_e^0) < M_m(0)$

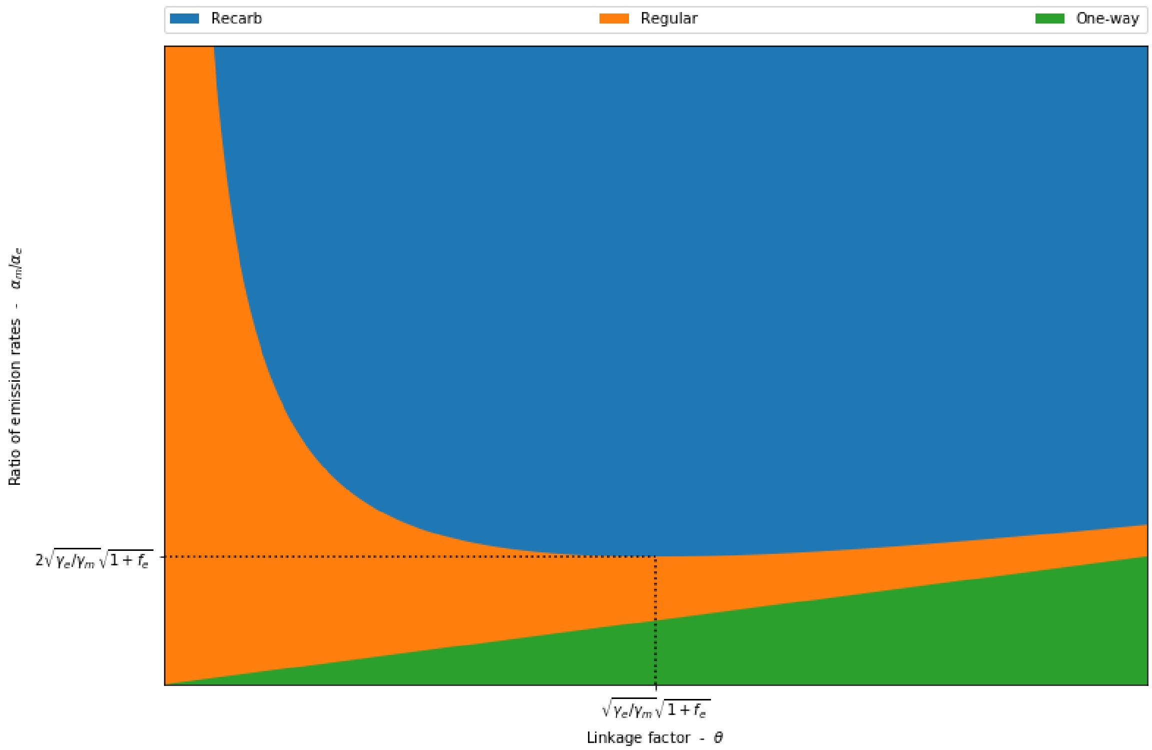

Figure 1: A map $(\theta, \frac{\alpha_m}{\alpha_e})$ representing the three possible regimes (Regular, Recarb and One-way).

Several comments can be made from this proposition. Unlike proposition 2, this proposition shows different regimes which determine the possible transition pathways. These three regimes are represented in Figure 1.

First regime, **One Way**, is for $\theta > \frac{\alpha_m}{\alpha_e}$. Here, the clean technology of the downstream sector requires so much input from the upstream sector that its ends up increasing total pollution unless the upstream sector is fully clean. The interpretation in term of life-cycle assessment would be that in-place dirty technologies should not be replaced with technologies whose global footprint, whichever its sectoral carbon footprint.

The second regime **Regular** occurs for $\theta < \frac{\alpha_m}{\alpha_e}$ and $\theta(x-\theta)\frac{\gamma_m}{\gamma_e} < 1 + f_e$. Under this regime, six possibles pathways with successive states each.

The third regime **Recarb** happens when $\theta(x-\theta)\frac{\gamma_m}{\gamma_e} < 1 + f_e$. It corresponds to the Proposition **??** under our specification. This regime differ with the previous one. First, in this regime, the clean investment in upstream clean technology can not follow the pace of the downstream transition. Said differently, as the marginal cost of the clean upstream technology may explode with the increase of its sector due to the transition of downstream, it may be optimal to invest in upstream dirty technology. This leads to two important differences. First the upstream sector can not ends its transition during the transition of the downstream sector. Second, when the downstream transition is included in the upstream transition, their may be transient decarbonization. This would happen for instance when the upstream initial sector size is sufficiently small.

Propositions 2 and 3, although some differences between regimes, have completely related pathways, ie the sequence of thresholds $\underline{\mu}_i, \bar{\mu}_i$ to a measure of effective abatement costs which combines sectoral abatement costs that include both interactions effect and the elasticity of sectoral demands. This result related to the traditional interpretation of MACs, stating that transition pathways follows the merit order of abatement costs.

3.3 Comparative statics

The previous section showed how the transition pathways were related to an effective measure of abatement costs . As the expression of these costs are complex, tt is worthy to determine how the main parameters of the problem may shift from a pathways to an other pathway. In this section will focus on sector sizes Q_i^0 (initial size with no carbon price). To do this, we derive the expression of the SCC thresholds (see appendix) and observe signs.

Figure 3.3 shows the effect of the increases of sector sizes Q_i^0. For the downstream sector, the increase of sector size only increases the MAC corresponding to full downstream decarbonization. As it ends up increasing the size of the upstream sector, it may also increase the price of the upstream sector good and therefore amplify the increase of the MAC of the downstream sector. Similarly, when the size of the upstream sector increases, only the pathways where downstream ends after upstream ends and start after upstream can

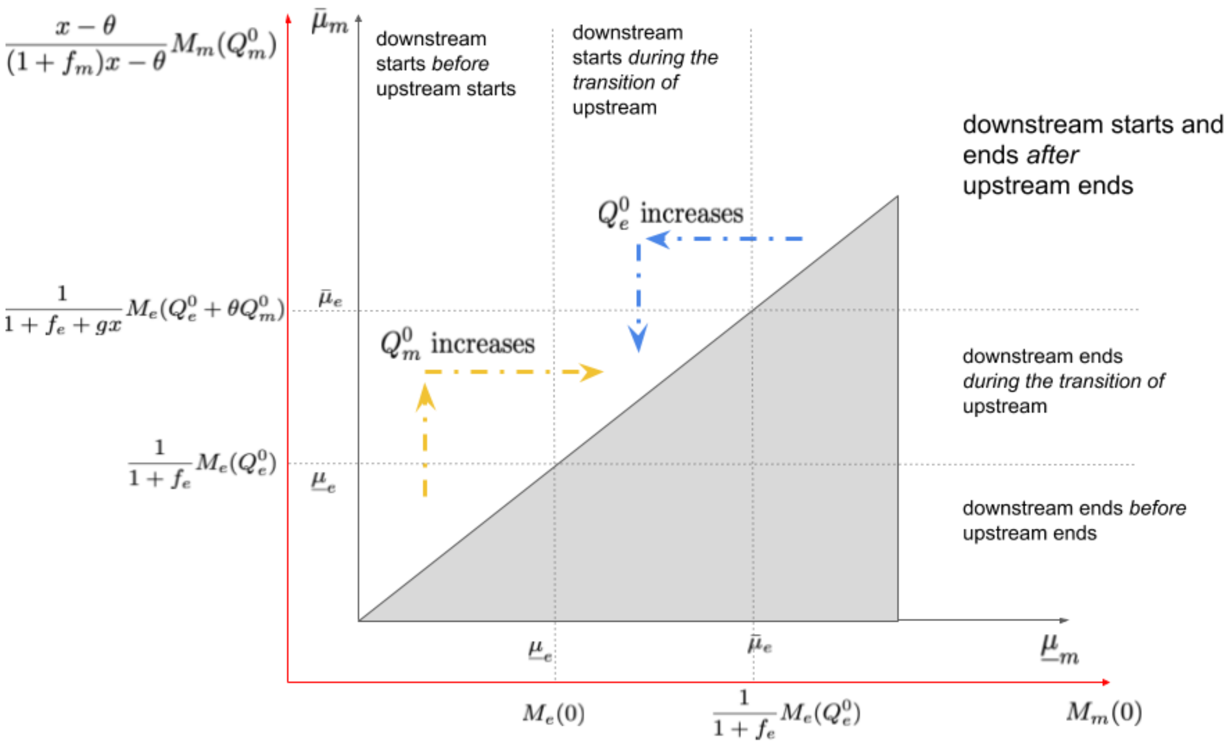

Figure 2: Impact of increasing sector sizes Q_i^0.

be modified.

Similar comparative statics can also be made for the other meaningful parameters, such as demand elasticities b_i and the ratio of emission rates $\frac{\alpha_m}{\alpha_e}$.

Of particular interest is the **Recarb** regime, the size of the downstream sector can have counterintuitive influence on the optimal sequencing. An increase of the size of downstream sector will delay the decarbonization of the upstream sector.

Corollary 1 *If $\theta(x-\theta)\frac{\gamma_m}{\gamma_e} > 1 + f_e$ holds and $M_e(0) < M_m(0)$, a small downstream sector ands after the upstream sector, while a large downstream sector ends before it..*

3.4 MAC curves

With our specification sectoral curves are linear encompassing both the reduction of the demand together with the progressive deployment of the clean technology.

The linkage between the two sector implies that the ”potential” of the upstream sector increases with the deployment of the downstream clean technology. In a sense baseline emissions are increasing in the upstream sector because of upstream decarbonization. And the MAC of the clean downstream progressively increase together with the price of the upstream good.

A parallel could be done between the “economist MAC” and LCA. There are two ways to write the equalization between the SCC and MAC in the downstream sector. First, if both technologies are used in the downstream sector then

$$\mu = \frac{1}{\alpha_m} \left[C'_{mc} + \theta P_e^* - C'_{md}\right]$$

and if the dirty technology is used in the upstream sector, the price P_e^* also encompasses the CSS since $P_e^* = C'_{md} + \alpha_e \mu$, and injecting this equation into the above one gives :

$$\mu = \frac{1}{\alpha_m - \theta\alpha_e} \left[C'_{mc} + \theta C'_{ed} - C'_{md}\right]$$

which would correspond to the LCA approach. The difference between being whether the SCC is already accounted for in the cost of the marginal upstream unit.

Figure, 3 illustrate two pathways in which one should start with the upstream sector. Notably a transient recarbonisation situation as described in Proposition 3, along which the upstream sector is fully clean before being recarbonized and eventually clean again.

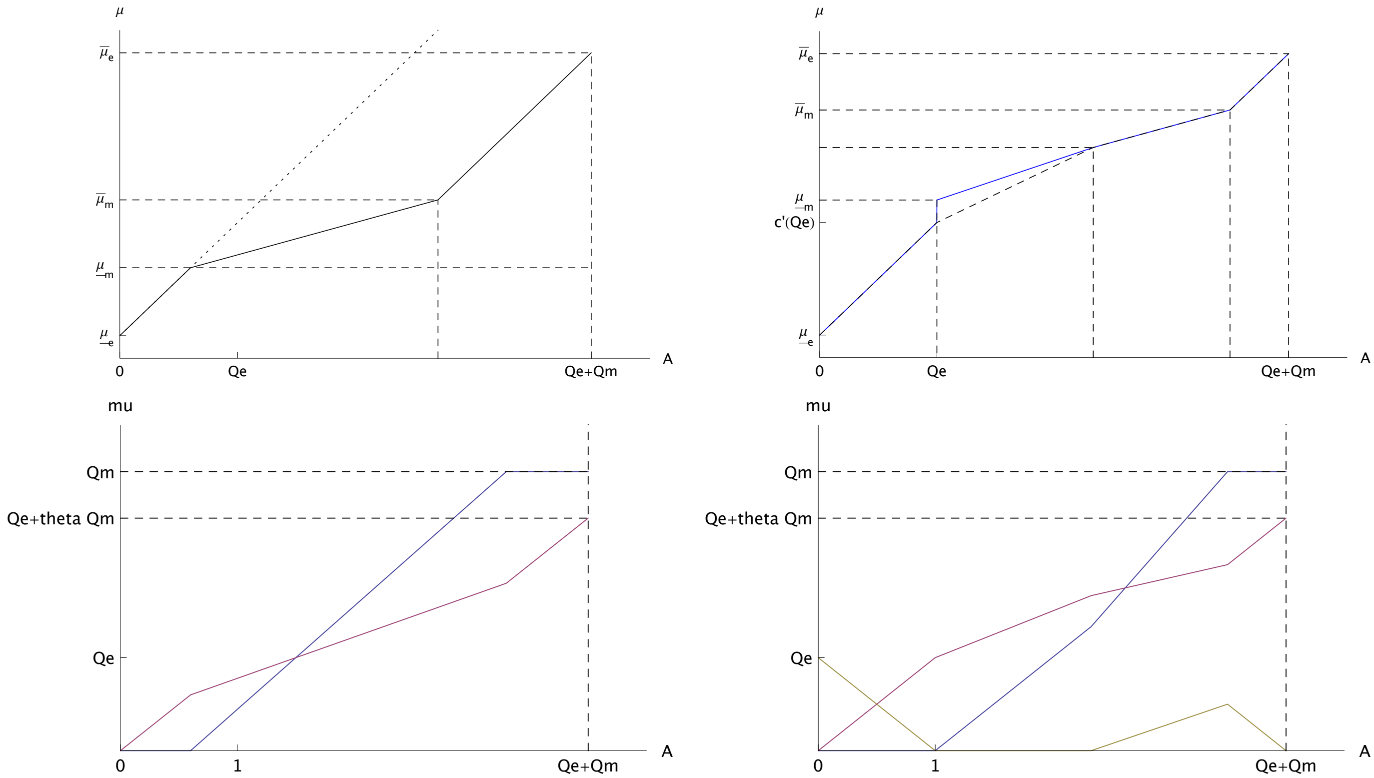

Figure 3: Two illustrative MAC-curve, with corresponding production. The blue line correspond to upstream clean production (e.g. electric vehicle), the purple one to clean upstream (e.g. renewable) and the brown to dirty upstream (e.g. coal power).

4 Policy coordination with imperfect carbon pricing

The previous part showed that a social planner integrates the effects of sector linkage in term of indirect costs and emissions from the downstream technology. In this section, we analyze the policy implications of sectoral linkage in relevant configurations. In particular, we investigate whether there is a need to coordinate policy instruments between sectors. We need a sufficiently flexible framework to consider various combinations of policy instruments to describe the varieties of real-world situations. It consists in a simple market equilibrium model with competitive firms in both sectors, price taking consumers. The regulator optimize welfare by setting policy instrument levels. Each sector is regulated with a price instrument or a quantity instrument. Price instruments includes both taxes on dirty productions and subsidies on clean productions. Quantity instruments are either quotas on minimum production by the clean technology, or quotas on maximum production by the dirty technology, or standard on the minimum share of clean technology utilization.

Definition 2 ***Policy coordination***

- *There is* <u>*downstream policy coordination*</u> *if the optimal downstream policy instrument* T_m^* *depends on the upstream instrument* T_e*:* $\frac{\partial T_m^*}{\partial T_e} \neq 0$
- *There is* <u>*upstream policy coordination*</u> *if the optimal upstream policy instrument* T_e^* *depends on the downstream instrument* T_m*:* $\frac{\partial T_e^*}{\partial T_m} \neq 0$

The direction of the policy coordination is the sign of $\frac{\partial T_i^*}{\partial T_j}$.

First we analyze the market equilibrium with price instruments in each sector and describe the first-best policy. Second we compute the optimal second-best subsidies when pigovian taxes are unavailable. Third, we describe the coordination between each couple of sectoral policies. Fourth, we modify our framework by including an emission cap instead of a exogenous social carbon cost. Fifth, we discuss and compare the effect of uncertainty on linkage intensity on welfare between the different policy settings.

4.1 Market equilibrium and Pigovian taxation

Let us denote P_e and P_m the consumers price of the upstream and downstream goods, t_i the tax on dirty units and s_i the subsidy on clean units of good $i = e, m$, both can indeed be negative. The market equilibrium is obtained with two representative consumers and a representative firm, all being price takers.In each sector $i = e, m$, a representative consumer maximizes the net surplus $S_i(Q_i) - P_i Q_i$, and the representative firm maximizes

$$\Pi(t, s, P_e, P_m, \mathbf{q}) = \sum_i [P_i(q_{id} + q_{ic}) - t_i q_{id} + s_i q_{ic} - C_{id}(q_{id}) - C_{ic}(q_{ic})] - \theta P_e q_{mc} \qquad (12)$$

subject to positivity constraints $q_{ij} \geq 0$ and $q_{ed} + q_{ec} \geq \theta q_{mc}$. Let us denote $q_{ij}^E(t, s)$ the equilibrium quantities, these satisfy:

$$S_i' = P_i = C_{id}'(q_{id}) + t_i - \psi_{id} \qquad \text{for } i = m, e \qquad (13)$$

$$C_{ec}'(q_{ec}) = P_e + s_e + \psi_{ec} \text{ and } C_{mc}'(q_{mc}) = P_m - \theta P_e + s_m + \psi_{ec} \qquad (14)$$

in which ψ_{ij} is the Lagrange multiplier of the positivity constraint $q_{ij} \geq 0$.

Lemma 3 *With a Pigovian tax, first-best can be decentralized and there is no need for policy coordination:* $t_i = \alpha_i \mu$ *and* $s_i = 0$

4.2 Second best subsidies

Taken as given the regulation of the upstream sector and the tax on the dirty downstream technology, one can determine the optimal downstream subsidy. That subsidy is justified by the unpriced negative externality from the dirty downstream technology but is influenced by the regulation of the upstream sector. The unpriced negative externality from the dirty upstream technology and the subsidy of the clean upstream technology both advocate for a reduction of the downstream subsidy.

Proposition 4 *For a triple* t_e, t_m *and* s_e, *the optimal subsidy on downstream production satisfies:*

$$s_m = \left[\sum_i (t_i - \alpha_i \mu) \frac{dq_{id}^E}{ds_m} - s_e \frac{dq_{ec}^E}{ds_m} \right] / \left[\frac{dq_{mc}^E}{ds_m} \right] \qquad (15)$$

With positive equilibrium quantities for each technology of both goods, it is

$$s_m = (\alpha_m \mu - t)\frac{1}{1+\epsilon_m} - \theta\Gamma\left[(\alpha_e\mu - t_e)\frac{1}{C''_{ed}} + s_e\frac{1}{C''_{ec}}\right] \tag{16}$$

with $\Gamma = [1/C''_{ed} + 1/C''_{ec} - 1/S''_e]^{-1}$ *and* $\epsilon_m = C''_{md}/(-S''_m)$

Corollary 2 *If dirty costs are linear the optimal subsidy downstream is*

$$s_m = \begin{cases} [\alpha_m\mu - t_m] - \theta[\alpha_e\mu - t_e] & \text{if } q_{ed} > 0 \\ [\alpha_m\mu - t_m] - \theta s_e & \text{if } q_{ed} = 0 \end{cases} \tag{17}$$

The optimal downstream subsidy is justified by unpriced externality, indeed, if the externality is taxed at the Pigovian level, so $t_i = \alpha_i\mu$ and $s_e = 0$, the optimal subsidy is null. In the downstream sector, an increase of clean production reduces dirty production by an amount determined by the slopes of consumer demand and of dirty marginal cost. If either the demand is inelastic or dirty cost are linear the rate of substitution is equal to minus one. The formula could be generalized to take into consideration that dirty and clean downstream goods are not perfect substitute on the consumer side(Xing et al., 2019).

Concerning the influence of the upstream sector regulation : First, if the externality is perfectly priced in the upstream sector, ($t_e = \alpha_e\mu$, $s_e = 0$) the emission intensity of the upstream sector does not intervene in the formula. It is so because the environmental cost is already encompassed in the upstream price. Second, the optimal downstream subsidy does not depend on the average mix in the upstream sector but on the emission intensity of the marginal unit which is a weighted sum of dirty and clean production, the weights depending on the slope of the respective marginal costs. With a linear dirty cost that marginal unit is dirty as long as there is some dirty production.

Indeed, the formula can be rewritten with elasticities of demand and supplies

Proposition 5 *For given taxes* t_m *and* t_e*, the optimal two subsidies* s_m *and* s_e *are*

$$s_m = [\mu\alpha_m - t_m]\frac{1}{1+\epsilon_m} - \theta s_e \tag{18a}$$

$$s_e = [\mu\alpha_e - t_e]\frac{1}{1+\epsilon_e} \tag{18b}$$

with $\epsilon_i = C''_{id}/(-S''_i)$*, which is null if the dirty production cost is linear or demand inelastic.*

Corollary 3 *For given taxes t_m and t_e, if demand functions are inelastic the first best can be decentralized with a couple of subsidies*

$$s_m = [\mu\alpha_m - t_m] - \theta[\mu\alpha_e - t_e] = \mu(\alpha_m - \theta\alpha_e) - (t_m - \theta t_e) \tag{19a}$$

$$s_e = \alpha_e\mu - t_e \tag{19b}$$

4.3 Coordination with alternative instruments

Other instruments are commonly used by policy makers to regulate emissions. This subsection aims at generalizing the previous result of second-best subsidy and policy coordination with different sets of sectoral instruments that include quantity instruments. We define standard and quotas as follow:

- Quotas on dirty production $\bar{q}_{id}$ such that $q_{id} \leq \bar{q}_{id}$
- Quotas on clean production $\bar{q}_{ic}$ such that $q_{ic} \geq \bar{q}_{ic}$
- Standard on clean production r_i such that $q_{ic} \geq r_i(q_{ic} + q_{id})$

The following results come from similar reasoning to the previous section and is provided in the appendix.

Proposition 6 *For the second-best downstream subsidy, there is downstream coordination for upstream tax and subsidies, standard and quotas on clean production, while there is no downstream coordination for upstream quotas on dirty production.*

Corollary 4 $\frac{\partial s_m^*}{\partial T_e} > 0$ *with T_e the upstream tax or standard.*

Proposition 7 *For the second-best upstream subsidy, there is upstream coordination for downstream tax and subsidy, and standard iif $C''_{ed} > 0$, while there is no upstream coordination for downstream quotas.*

4.4 Coordination with endogenous social carbon cost

We now consider a given carbon budget $\bar{E}$ and analyze the choice of instruments The regulator program becomes:

$$\max_{s_e,s_m} \sum_i S_i(Q_i) - P_i Q_i + \Pi(t, s, P_e, P_m, \mathbf{q})$$

s.t

$$\alpha_e q_{ed} + \alpha_m q_{md} \leq \bar{E} \tag{20}$$

Where quantities Q_i, q_{ij} are determined by market equilibrium conditions. For simplicity, we consider linear dirty technology costs for each sector. The Lagrange multiplier of the emission cap condition μ can be considered as the social cost of carbon. Hence with this framework, second-best subsidies have the same form as given by equations 17. However, this SCC is now endogenous and depends on policy instruments levels. Under taxes t_e, t_m, the emission cap constraints allows

Simple comparative statics gives the variations of μ:

$$\frac{\partial \mu^{SB}}{\partial t_e} = \frac{\frac{\alpha_e}{S_e''}}{\frac{\alpha_e^2}{C_{ec}''} + \frac{(\alpha_m - \theta\alpha_e)^2}{C_{mc}''}} < 0 \tag{21a}$$

$$\frac{\partial \mu^{SB}}{\partial t_m} = \frac{\frac{\alpha_m}{S_m''}}{\frac{\alpha_e^2}{C_{ec}''} + \frac{(\alpha_m - \theta\alpha_e)^2}{C_{mc}''}} < 0 \tag{21b}$$

From these variations, we are able to compute the variations of the second-best subsidies: $\frac{\partial s_i^*}{\partial t_i} < 0$. Interestingly, we have the following variations:

$$\frac{\partial s_e^*}{\partial t_m} < 0 \tag{22a}$$

$$\frac{\partial s_m^*}{\partial t_e} \propto (\alpha_m - \theta\alpha_e)\Big(\frac{\theta(\alpha_m - \theta\alpha_e)}{C_{mc}''} - \alpha_e(\frac{1}{C_{ec}''} - \frac{1}{S_e''})\Big) + \frac{\alpha_m\alpha_e}{C_{ec}''} \lesseqgtr 0 \tag{22b}$$

This can be summarized as follows:

Proposition 8 *Under endogenous social carbon cost:*

- *There is upstream coordination with downstream tax and subsidy.*

- *The downstream coordination have an ambiguous direction.*

Corollary 5 *Recarb is a sufficient condition to have* $\frac{\partial s_m}{\partial t_e} > 0$

4.5 Coordination under uncertain linkage intensity

Until here, we assumed that the the linkage intensity θ was certain. However, this parameter main be difficult to estimate. It requires life-cycle analysis which may be very sensitive to methodologies and data sources used. As a result, one may think about the current debate on the carbon footprint of electric vehicles. Such consideration legitimate the idea that a regulator may be uncertain on the intensity. In the goal of this subsection is to assess the effectiveness of the different .

We assume now that θ is random variable with mean $\bar{\theta}$ and variance $Var(\theta)$. In general, the welfare effect from uncertainty can be deduced from a simple taylor expansion:

$$E[W] = W(\bar{\theta}) + \frac{1}{2}Var(\theta)\frac{\partial^2 W}{\partial \theta^2}\Big|_{\theta=\bar{\theta}} + \cdots \tag{23}$$

Moreover, in order to obtain tractable results, we will use the specified version of the model, with linear demand, linear dirty costs and quadratic clean costs.

We find :

- Downstream tax-subsidy: $\frac{\partial^2 W}{\partial \theta^2} = \gamma_m\big((\alpha_e\mu + c_{ed})^2 - (\alpha_e\mu - t_e)^2\big)$

- Downstream standard: $\frac{\partial^2 W}{\partial \theta^2} = \frac{r_m^2\gamma_m}{r_m^2+\gamma_m b_m}\big((\alpha_e\mu + c_{ed})^2 - (\alpha_e\mu - t_e)^2\big)$

- Downstream dirty quotas: $\frac{\partial^2 W}{\partial \theta^2} = \frac{\gamma_m}{1+\gamma_m b_m}\big((\alpha_e\mu + c_{ed})^2 - (\alpha_e\mu - t_e)^2\big)$

- Downstream clean quotas: $\frac{\partial^2 W}{\partial \theta^2} = 0$

Proposition 9 *The uncertainty on the linkage uncertainty affects welfare differently according to the downstream instrument. These effects can be ordered as follows:*
Tax-Subsidy > Dirty Quota > Standard > Clean Quota

5 Conclusion

Our simple model allowed us to consider transition in a economy with interconnected sectors. We established that it is indeed optimal to shift emissions from a downstream to an upstream sector, and along an optimal trajectory upstream emissions can well be increasing because of that induced demand.

We showed that Marginal Abatement Costs Curves could be easily corrected by integrating sectoral interactions. However, the endogeinity between the downstream and upstream transition could make MACCs more difficult to interpret as it could hide transient decarbonization (even with a quadratic simple specification).

The analysis of second-best subsidy in the downstream sector stressed three main points: only unpriced externalities influence the optimal subsidy, the marginal upstream unit and not the average one influence the optimal downstream subsidy, along a decarbonization transition the optimal downstream subsidy should evolved depending on the state of the upstream sector.

This work could be improved in several ways. First our model could be extended to an economy with a more sectors a with more complex structure. This would allow to build real-world MAC curves based on Input-Output Matrices. Second, our second-best analysis could be applied to a situation where different regulators (different agencies, federal/state regulators) would decide based on their own objectives. A close topic could be the situation where

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A Proof of Proposition 1

In configuration B.

Preliminaries

We use sectoral surpluses to analyze the demand $D(P_e, \mu)$ of the upstream good emanating from the downstream sector, and $O(P_e, \mu)$ the supply from the upstream sector (the difference between total production and consumption). Indeed, Roy's identity:

$$D(P_e, \mu) = \theta q_m = -\frac{\partial V_m}{\partial P_e} \text{ and } O(P_e, \mu) = q_{ed} + q_{ec} - Q_e = \frac{\partial V_e}{\partial P_e}$$

At the optimum P_e^* solves $D(P_e, \mu) = O(P_e, \mu)$

- Demand from downstream: the first order conditions are

$$S'_m(q_{md} + q_{mc}) - C'_{md}(q_{md}) - \alpha_m \mu = 0 \quad (24)$$

$$S'_m(q_{md} + q_{mc}) - C'_{mc}(q_{mc}) - \theta P_e = 0 \quad (25)$$

taking the derivatives with respect to μ gives:

$$\begin{bmatrix} \frac{\partial q_{md}}{\partial \mu} \\ \frac{\partial q_{mc}}{\partial \mu} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} S''_m - C''_{mc} & -S''_m \\ -S''_m & S''_m - C''_{md} \end{bmatrix} \begin{bmatrix} \alpha_m \\ 0 \end{bmatrix} \quad (26)$$

and with respect to the upstream price P_e:

$$\begin{bmatrix} \frac{\partial q_{md}}{\partial P_e} \\ \frac{\partial q_{mc}}{\partial P_e} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} S''_m - C''_{mc} & -S''_m \\ -S''_m & S''_m - C''_{md} \end{bmatrix} \begin{bmatrix} 0 \\ \theta \end{bmatrix} \quad (27)$$

with $\Delta = -S''_m(C''_{md} + C''_{mc}) + C''_{md}C''_{mc}$ so

$$\frac{\partial q_{mc}}{\partial \mu} = \alpha_m \frac{-S''_m}{\Delta} \text{ and } \frac{\partial q_{mc}}{\partial P_e} = -\theta \frac{C''_{md} - S''_m}{\Delta} \tag{28}$$

and

$$\frac{\partial q_{md}}{\partial \mu} = -\alpha_m \frac{C''_{mc} - S''_m}{\Delta} \text{ and } \frac{\partial q_{md}}{\partial P_e} = \theta \frac{-S''_m}{\Delta} \tag{29}$$

- Supply from upstream: situation is much simpler since $q_{ed} = C'^{-1}_{ed}(P_e - \alpha_e \mu)$, $q_{ec} = C'^{-1}_{ec}(P_e)$ and $Q_e = S'^{-1}_e(P_e)$ so that

$$O(P_e, \mu) = C'^{-1}_{ed}(P_e - \alpha_e \mu) + C'^{-1}_{ec}(P_e) - S'^{-1}_e(P_e)$$

and

$$\frac{\partial O}{\partial \mu} = -\frac{\alpha_e}{C''_{ed}} \text{ and } \frac{\partial O}{\partial P_e} = \frac{1}{C''_{ed}} + \frac{1}{C''_{ec}} + \frac{1}{-S''_e} \tag{30}$$

- $P^*_e(\mu)$ solves $O(\mu, P_e) = D(\mu, P_e)$ so that

$$\frac{dP^*_e}{d\mu} = \frac{\partial D/\partial \mu - \partial O/\partial \mu}{\partial O/\partial P_e - \partial D/\partial P_e} > 0 \tag{31}$$

Proof of Lemma 1

Proof of Proposition1:

In configuration B, the quantity of dirty upstream increases if and only if $P^*_e(\mu) - \alpha_e \mu$ increases that is, from eq. (31):

$$\frac{\partial D}{\partial \mu} - \frac{\partial O}{\partial \mu} > \alpha_e \left[\frac{\partial O}{\partial P_e} - \frac{\partial D}{\partial P_e}\right] \Leftrightarrow \frac{\partial D}{\partial \mu} + \alpha_e \frac{\partial D}{\partial P_e} > \alpha_e \frac{\partial O}{\partial P_e} + \frac{\partial O}{\partial \mu}$$

injecting the expressions (29) and (30) for derivatives obtained

$$\theta \left[\alpha_m \frac{-S''_m}{\Delta} - \alpha_e \theta \frac{C''_{md} - S''_m}{\Delta}\right] > \alpha_e \left[\frac{1}{C''_{ed}} + \frac{1}{C''_{ec}} + \frac{1}{-S''_e}\right] - \frac{\alpha_e}{C''_{ed}} = \alpha_e \left(\frac{1}{C''_{ec}} + \frac{1}{-S''_e}\right)$$

then using the expression of Δ and dividing by $-S''_m$ one obtains inequality (10)

B Alternative instruments

B.1 Optimal subsidy for downstream

B.1.1 Upstream sector with mandate

Market Equilibrium

$$U_e'(Q_e) = \langle c_e(Q_e + \theta q_{mc}) \rangle$$
$$U_m'(Q_m) = C_{md}'(Q_m - q_{mc}) + t_m = C_{mc}'(q_{mc}) + \theta U_e' - s_m$$

With $\langle c_e(q_e) \rangle = r_e C_{ec}'(r_e q_e) + (1 - r_e) C_{ed}'((1 - r_e) q_e)$

Comparative statics

$$(V_e - U_e'') dQ_e = -\theta V_e dq_{mc}$$
$$(C_{md}'' - U_m'') dq_{md} = -U_m'' dq_{mc}$$

Where $V_e = r_e^2 C_{ec}'(r_e Q_e) + (1 - r_e)^2 C_{ed}''$

Second-best subsidy

$$s_m = \frac{1}{1 - \frac{C_{md}''}{U_m''}} (\alpha_m \mu - t_m) - \frac{1 - r_e}{1 - \frac{V_e}{U_e''}} \theta \alpha_e \mu$$

B.1.2 Upstream sector with quotas on dirty

$$U_e'(Q_e) = C_{ec}'(Q_e + \theta q_{mc} \bar{q}_{ed}) \langle$$
$$U_m'(Q_m) = C_{md}'(Q_m - q_{mc}) + t_m = C_{mc}'(q_{mc}) + \theta U_e' - s_m$$

Comparative statics

$$(C_{ec}'' - U_e'') dQ_e = -\theta C_{ec}'' dq_{mc}$$
$$(C_{md}'' - U_m'') dq_{md} = -U_m'' dq_{mc}$$

Second-best subsidy

$$s_m = \frac{1}{1 - \frac{C''_{md}}{U''_m}}(\alpha_m \mu - t_m)$$

B.1.3 Upstream sector with quotas on clean

$$U'_e(Q_e) = C'_{ed}(Q_e + \theta q_{mc} - \bar{q}_{ec})\langle$$
$$U'_m(Q_m) = C'_{md}(Q_m - q_{mc}) + t_m = C'_{mc}(q_{mc}) + \theta U'_e - s_m$$

Comparative statics

$$d =$$
$$(C''_{md} - U''_m)dq_{md} = -U''_m dq_{mc}$$

Second-best subsidy

$$s_m = \frac{1}{1 - \frac{C''_{md}}{U''_m}} - \frac{\theta\mu}{1 - \frac{C''_{ed}}{U''_e}}$$

B.2 Optimal subsidy for upstream

B.2.1 Downstream sector with mandate

$$U'_e(Q_e) = C'_{ed}(Q_e + \theta r_m Q_m - q_{ec}) + t_e = C'_{ec}(q_{ec}) - s_e$$
$$U'_m(Q_m) = \langle c_m(Q_m)\rangle + r_m \theta U''_e$$

Comparative statics

$$\chi dQ_e = F_m C''_{ed} dq_{ec}$$
$$\chi dQ_m = -\theta r_m C''_{ed} U''_e dq_{ec}$$

Where $F_m = r_m^2 C'_{mc}(r_m Q_m) + (1-r_m)^2 C''_{md} - U''_m$, $F_e = C''_{ed} - U''_e$, $\chi = F_e F_m - (r_m\theta)^2 U''_e C''_{ed}$

Second-best subsidy

$$s_e = (\alpha_e \mu - t_e)\big(1 - \frac{F_m C'_{ed}}{\chi}\big) + \mu\big((1-r_m)\alpha_m + \theta r_m \alpha_e\big)\frac{r_m \theta U''_e C''_{ed}}{\chi}$$

B.2.2 Downstream sector with quotas on dirty

$$U'_e(Q_e) = C'_{ed}(Q_d - q_{ec} + \theta(Q_m - \bar{q}_{md})) + t_e = C'_{ec}(q_{ec}) - s_e$$
$$U'_m(Q_m) = C'_{mc}(Q_m - \bar{q}_{md}) + \theta U_e(Q_e)$$

Comparative statics

$$\chi dQ_e = F_m C''_{ed} dq_{ec}$$
$$F_m dQ_m = -\theta U''_e dQ_e$$

Where $F_m = C''_{mc} - U''_m$, $F_e = C''_{ed} - U''_e$, $\chi = F_e F_m - (r_m\theta)^2 U''_e C''_{ed}$

Second-best subsidy

$$s_e = (\alpha_e\mu - t_e)\big(1 - \frac{F_m - \theta^2 U''_e)C''_{ed}}{\chi}\big) = (\alpha_e\mu - t_e)\big(\frac{-U''_e F_m}{\chi}\big)$$

B.2.3 Downstream sector with quotas on clean

$$U'_e(Q_e) = C'_{ed}(Q_d - q_{ec} + \theta\bar{q}_{mc}) + t_e = C'_{ec}(q_{ec}) - s_e$$
$$U'_m(Q_m) = C'_{md}(Q_m - \bar{q}_{mc})$$

Comparative statics

$$F_e dQ_e = C''_{ed} dq_{ec}$$
$$F_m dQ_m = 0$$

Second-best subsidy

$$s_e = \frac{1}{1 - \frac{C''_{ed}}{U''_e}}(\alpha_e\mu - t_e)$$