Fully-Funded Clean Technology Support

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Abstract

This paper analyzes fully-funded schemes aimed at supporting clean production—ranging from a user tax, a producer tax, and a tradeable quota, to a standard of production. With no income taxes, each of the equally welfare-ranked schemes fails to implement efficient joint dirty and clean production, because clean goods are overpriced relative to their marginal benefit when both clean and dirty goods are produced, however implements efficient pure clean production, because the subsidy can push dirty goods out of the market. Generally, with elastic or inelastic labor supply and labor income taxes, a two-part tariff helps to achieve efficiency. *(JEL D62, H23, Q58)*

1 Introduction

Mounting evidence of adverse effects by fossil fuel use onto the climate and continued lack of success in finding a safe storage site for waste from nuclear power have led to the promotion of renewable energy production with the help of schemes that fully cover the cost of clean production in the market, or fully-funded schemes, in many jurisdictions in the world. This makes clean production economically viable to private users, and alleviates conflicts of interest over redistribution that otherwise arise from taxing emissions or setting emissions quotas. The environmental problems of climate change and nuclear waste have thus rung in a paradigm shift in energy policy with introducing new policies. This paper attempts to better understand how these schemes also discourage the use of dirty energy technology, their welfare consequences, and the optimal design of fully-funded clean technology support schemes with an environmental motive in a systematic way.

* *Keywords*: Privately provided public goods, Climate change, Nuclear energy, Renewable energy support.

For these purposes, I develop a general equilibrium model. Time can be used as labor for producing a consumption good with the use of dirty or clean technology or enjoyed as leisure. Dirty production adversely affects the economy (impacting utility or production) which private agents ignore, creating a pollution externality. As regulatory schemes supporting clean technology are balanced in a way that permits payments administered only by private agents, hence do not require direct subsidies from a government budget, the model features no government budget constraint. In the model, the clean and dirty goods can be perfect or imperfect substitutes. To evaluate the welfare of support schemes, I closely follow the primal approach to optimal taxation in Chari & Kehoe (1998).

The paper's main results can be grouped as follows.

One, a price system and regulatory policy can be defined in multiple ways yielding the same allocation within a given scheme, hence the *policy space* with a given fully-funded clean technology support scheme is indeterminate. Important to formulate an equilibrium, a support scheme can be made up of price and policy as will be used. Obviously, the value of a certificate constitutes a price and an intensity requirement can be a policy in a given scheme (quota). In another example, a support scheme consists of only policy, each a specific tax and subsidy (feebate). Less obvious, prices and a policy in the form of a subsidy form a scheme (general tax). The tax then appears implicit. The tax, when made explicit, and the subsidy then can be understood as a surcharge and a premium. I also generalize to cases between buy-out (tariff) and premium depending on the access of clean supply to a wholesale market. I thus obtain an *equivalency result about welfare*. Each welfare-maximizing policy in a fully-funded scheme of general funding (from buy-out to premium with general tax, or feebate) and quota (with intensity or percentage requirement of clean goods)yields the same constrained efficient or efficient welfare, because each scheme yields the same implementability constraint. Surprisingly, for implementation of allocations it does not matter if one taxes both dirty and clean energy or only dirty energy to fund a subsidy to clean energy, because between these general and specific funding schemes the subsidy adjust in a way to produce an identical implementability constraint. Similarly, for implementation it does not matter if using an intensity or portfolio quota, because the requirement and certifcate price adjust to yield the same implementability constraint.

Two, fully-funded clean technology support *cannot achieve efficiency with* *dirty production* unless they are equipped with a two-part tariff. The reason is that a fully-funded scheme supporting some technology equates the marginal benefit to a weighted average of marginal cost of consumption using goods from different technologies. Efficiency, however requires marginal benefit and cost for a nonexternality-generating technology to be equal. Fully-funded clean technology support *achieves efficiency with no dirty production.* This occurs, because the conditions for efficiency and equilibrium coincide when efficient allocations are characterized by no dirty production, which includes cases with and with no societal externality cost at the margin of zero dirty output. A two-part tariffs tags quantity to price, and thereby can equate marginal benefit and cost for a nonexternality-generating technology.

Three, support of clean technology effectively lowers pollution, a property not shared with support of technology polluting less than the most productive technology.

1.A Background and Literature

Background on Renewable Energy Support.—The main starting points for fully-funded renewable energy support were fully-funded subsidies to solar PV in more than three dozen municipalities in Germany starting in 1989 and a federal law providing market access and remunerations for electricity from small-scale run-of-river hydro plants, klaer-, deponie-, und biogas, wind turbines, and solar PV modules from 1991 on in Germany. The municipalties as the owners of local electricity distributors had the political will for electricity prices to cover the cost of solar PV, and so the local electricity distributors asked the state commissions responsible for regulating electricity tariffs to start fully funding solar PV locally. The two key elements of full funding and market access were then combined in the federal law of the EEG providing renewable energy support since 2000.

Relationship to the Literature.—This paper synthesizes previous work by systematically studying different schemes of clean technology support and classifying them.

The literature sometime talks about revenue-neutral taxation which does not pay tribute to how the theory of renewable energy support has been borne out of practice.

2 A Model with One Dirty and One Clean Technology

This section presents and analyzes a model with a dirty and a clean technology useful to study clean technology promotion. This uses labor as an input in both technologies, and inelastic labor supply.

This section describes a model with one dirty and one clean production technology in a partial equilibrium setting. The model generally speaks to the support of producing goods using non-polluting, or clean, technology. The description of the model will be guided by the support of clean energy.

A continuum of identical consumers and producers each with size 1 populates the economy.

Tastes and Technology.—Consuming the quantity of services q yields the benefit $B : \mathbb{R}_+ \to \mathbb{R}$ increasing, twice differentiable, and strictly concave. Producing the quantities of a dirty and a clean good e_X and e_Y requires expending the cost $C_X : \mathbb{R}_+ \to \mathbb{R}$ and $C_Y : \mathbb{R}_+ \to \mathbb{R}$ increasing, twice differentiable, and concave, with an advantage of dirty relative to clean technology using the partial differentials $C'_X < C'_Y$ all $(e_X, e_Y) \in \mathbb{R}^2_+$. The dirty good creates emissions of equal magnitude, while the clean good does not create emissions, and emissions themselves cause damages $D : \mathbb{R}_+ \to \mathbb{R}$ increasing, twice differentiable, and convex. With productivity $z > 0$ the services are derived from energy,

$$q = z(e_X + e_Y). \tag{1}$$

Support Schemes.—A scheme supporting clean production can be defined in various ways with a list of prices and policy ϕ and π. The prices and policy affect the consumer unit expenditure and the producer unit revenue, which I will define for each scheme. Notice that prices are formed by the interplay of demand and supply schedules given policy a regulator sets. All schemes contain a subsidy.

2.A User Tax

The goal of each consumer is choosing consumption q so to maximize utility $B(q) - pq - D(e_X)$, taking as given the price p, and emissions e_X. A producer chooses (e, e_X, e_Y) so to maximize profits $\Pi = pze - (\varphi + \tau^C)e + \varphi e_X - C_X(e_X) + (\varphi + \sigma^C)e_Y - C_Y(e_Y)$, taking as given the prices (p, φ), the tax rate τ^C, and the subsidy rate σ^C. Thus, prices are given by $\phi = (p, \varphi, 1)$

including one for the normalized price of the input, while policy amounts to $\pi = (\tau^C, \sigma^C)$.

Consumer and producer behavior imply that:

$$
\begin{aligned}
& zB' = \varphi + \tau^C, \\
& \varphi \leq C'_X, \ \text{ with equality if } \ e_X > 0, \\
& \varphi + \sigma^C \leq C'_Y, \ \text{ with equality if } \ e_Y > 0.
\end{aligned}
\tag{2}
$$

Payment balance, which expresses that the tax ($\tau^C(e_X + e_Y)$) equals the subsidy ($\sigma^C e_Y$), requires that

$$
\tau^C e_X + (\tau^C - \sigma^C) e_Y = 0. \tag{3}
$$

A subsidy in the form of a premium thus covers the difference between the producer cost of clean energy for the marginal unit produced and the market price.

In Germany the fee, or surcharge ("EEG-Umlage"), is differentiated so that firms that use a large amount of electricity or that provide railway services pay a reduced fee. In practice, demand and supply, price, and subsidy rate determine the difference between the revenue and producer cost of clean energy aggregated over production units and a year that needs to be funds. This difference then is distributed over aggregate supply, so the fee appears as a residual.

2.B Producer Tax

The consumer decision problem is the same as with a user tax above. A producer chooses (e, e_X, e_Y) so to maximize profits $\Pi = pze - \varphi e + (\varphi - \tau^P)e_X - C_X(e_X) + (\varphi - \tau^P + \sigma^P)e_Y - C_Y(e_Y)$, taking as given the prices (p, φ), the tax rate τ^P, and the subsidy rate σ^P. Thus, prices are given by $\phi = (p, \varphi, 1)$, while policy amounts to $\pi = (\tau^P, \sigma^P)$.

Consumer and producer behavior imply that:

$$
\begin{aligned}
& zB' = \varphi, \\
& \varphi - \tau^P \leq C'_X, \ \text{ with equality if } \ e_X > 0, \\
& \varphi - \tau^P + \sigma^P \leq C'_Y, \ \text{ with equality if } \ e_Y > 0.
\end{aligned}
\tag{4}
$$

Payment balance, which expresses that the tax ($\tau^P(e_X + e_Y)$) equals the subsidy ($\sigma^P e_Y$), requires that

$$\tau^P e_X + (\tau^P - \sigma^P)e_Y = 0. \tag{5}$$

2.C Feebate

The decision by each household follows as with the scheme of the consumer tax. A producer chooses (e, e_X, e_Y) so to maximize profits $\Pi = pze - \varphi e + (\varphi - \tau^F)e_X - C_X(e_X) + (\varphi + \sigma^F)e_Y - C_Y(e_Y)$, taking as given the prices (p, φ), the tax rate τ^F, and the subsidy rate σ^F. Thus, prices are given by $\phi = (p, \varphi, 1)$, while policy amounts to $\pi = (\tau^F, \sigma^F)$.

Consumer and producer behavior imply that:

$$\begin{aligned} &zB' = \varphi, \\ &\varphi - \tau^F \leq C'_X, \text{ with equality if } e_X > 0, \\ &\varphi + \sigma^F \leq C'_Y, \text{ with equality if } e_Y > 0 \end{aligned} \tag{6}$$

Payment balance, which expresses that the tax ($\tau^F e_X$) equals the subsidy ($\sigma^F e_Y$), requires that

$$\tau^F e_X - \sigma^F e_Y = 0. \tag{7}$$

Here at the same time the tax and subsidy are called fee and rebate.

2.D Tradebale Intensity Quota

The decision by each household follows as with the scheme of the consumer tax. A producer chooses (e, e_X, e_Y) so to maximize profits $\Pi = pze - \varphi e + (\varphi - \gamma\lambda^N)e_X - C_X(e_X) + (\varphi + \gamma)e_Y - C_Y(e_Y)$, taking as given the prices (p, φ, γ), and the quota λ^O and policy $\pi = \lambda^N$.

Consumer and producer behavior imply that:

$$\begin{aligned} &zB' = \varphi, \\ &\varphi - \gamma\lambda^N \leq C'_X, \text{ with equality if } e_X > 0, \\ &\varphi + \gamma \leq C'_Y, \text{ with equality if } e_Y > 0 \end{aligned} \tag{8}$$

The quota requires that

$$\lambda^N e_X - e_Y = 0. \tag{9}$$

2.E Tradebale Portfolio Quota

The consumer decision problem is the same as with a user tax above. A producer chooses (e, e_X, e_Y) so to maximize profits $\Pi = pze - \varphi e + (\varphi - \gamma\lambda^O)e_X - C_X(e_X) + (\varphi + (1-\lambda^O)\gamma)e_Y - C_Y(e_Y)$, taking as given the prices (p, φ, γ), and the policy $\pi = \lambda^O$.

Consumer and producer behavior imply that:

$$
\begin{aligned}
& zB' = \varphi, \\
& \varphi - \gamma\lambda^O \leq C'_X, \ \text{ with equality if } \ e_X > 0, \\
& \varphi + (1-\lambda^O)\gamma \leq C'_Y, \ \text{ with equality if } \ e_Y > 0
\end{aligned}
\tag{10}
$$

The quota requires that

$$
\lambda^O(e_X + e_Y) - e_Y = 0. \tag{11}
$$

Different to the intensity quota, producers need to hold a certificate for each unit of clean good produced.

Standards can be formulated in an analogous way to quotas such that producers view a side constraint.

2.F Relationship Between Policies

Prices for schemes with tax/subsidy or standards are φ, and for schemes with tradeable quota (φ, γ).

The producer-sided policies can be summarized with the "tax rates" α_X and α_Y on the dirty and clean good, for producer cost, feebate, and tradeable portfolio and intensity quota,

$$
\alpha_X = \begin{cases} \tau^P \\ \tau^F \\ \gamma\lambda^N \\ \gamma\lambda^O \end{cases},
$$

$$
\alpha_Y = \begin{cases} \tau^P - \sigma^P \\ -\sigma^F \\ -\gamma \\ \lambda^O\gamma - \gamma \end{cases}.
$$

Table 1

Classification of Clean Technolgy Support Schemes

Type	Scheme
Tax	General Tax
	* User tax, Producer tax, Specific Tax
	* Feebate
Quota	Tradeable Quota
	* Intensity
	* Portfolio
	* Average Output
	Standard
	* Intensity
	* Portfolio
	* Average Emissions

Notes: The tax schemes use funding with a subsidy, while quota schemes contain funding with no subsidy. A tradeable quota or standard are viewed by producers here as such policies may be obeyed by consumers only if the dirty and clean goods are imperfect substitutes. A standard can be referred to as a nontradeable quota. Funding refers to revenue for clean output.

These expressions are useful to show the equivalency of support schemes with respect to welfare.

To indicate that the "tax rates" depend on policies, one may append the policies as arguments, that is, a producer maximizes profit $\Pi = (p - \alpha_X(\pi^j))e_X - C_X(e_X) + (p - \alpha_Y(\pi^j)) - C_Y(e_Y)$, taking as given the policies π^j and prices ϕ^j. The price for a quota itself depends on policy. Payment balance, which expresses that the tax $(\alpha_X(\pi^j)e_X)$ equals the subsidy $(-\alpha_Y(\pi^j)e_Y)$, can be stated as

$$\alpha_X(\cdot)e_X + \alpha_Y(\cdot)e_Y = 0. \tag{12}$$

2.G Implementation

I will now characterize allocations which are implementable with fully-funded support schemes and compare them to the optimal allocation.

Table 2

Policies

Scheme	Policy	Price
Tax		
Surcharge on Price ("User tax")	$(\tau^{\mathrm{C}}, \sigma^{\mathrm{C}})$	φ
Deduction from Price ("Producer tax")	$(\tau^{\mathrm{P}}, \sigma^{\mathrm{P}})$	φ
Tax on Dirty Production ("Feebate")	$(\tau^{\mathrm{F}}, \sigma^{\mathrm{F}})$	φ
Tradeable Quota		
Intensity Quota	λ^{N}	(φ, γ)
Portfolio Quota	λ^{O}	(φ, γ)
Green Offset	λ^{G}	(φ, γ)
Standard		
Intensity Standard	λ^{N}	$\varphi)$
Portfolio Standard	λ^{O}	φ
Average Emissions	s	φ

Notes: A tradeable quota or standard are viewed by producers here as such policies may be obeyed by consumers only if the dirty and clean goods are imperfect substitutes. A standard can be referred to as a nontradeable quota.

We can see that maximizing welfare $\mathrm{m}^{\mathrm{PE}}(q, e_X, e_Y) = B(q) - D(\psi e_X) - C_X(e_X) - C_Y(e_Y)$subject to (1) are allocations that satisfy the necessary social optimality conditions

$$\begin{aligned} zB' &\leq C'_X + \psi D', \text{ with equality if } e_X > 0, \\ zB' &\leq C'_Y, \text{ with equality if } e_Y > 0. \end{aligned} \tag{13}$$

The assumptions on the benefit function (B) and cost functions (C_X, C_Y, and D) imply the existence of a unique optimum given parameter values including the exposure to environmental damages. Thus for sufficiently low exposure joint dirty and clean production is optimal ($e_X > 0$, $e_Y > 0$) while for sufficiently large exposure pure clean production is optimal ($e_X = 0$, $e_Y > 0$).

With the clean technology support schemes stated above, decisions on the usage and production side of the economy can be described by:

Lemma 1. *In a competitive equilibrium with clean technology support*

$$(zB' - C'_X)e_X + (zB' - C'_Y)e_Y = 0. \tag{14}$$

The implementablity constraint (14) devises that the fully-funded support helps to open a gap between the marginal benefit and cost of dirty production which goes some way toward correcting the external cost of dirty production, $C'_X < B'$, whenever both the dirty and clean good are produced. However, it also creates a gap between the marginal cost and benefit of clean production, $B' < C'_Y$, whenever the dirty good becomes produced. Condition (14) key in characterizing fully-funded clean technology support holds with joint dirty and clean production and pure clean production.

PROPOSITION 1 (i). *Clean technology support with a fully-funded scheme cannot implement an efficient allocation with dirty production, $e_X > 0$.*

Efficiency requires that the marginal benefit exceeds the marginal cost for dirty production, $zB' > C'_X$, as that creates a negative externality, and equals the marginal cost of clean production, $zB' = C'_Y$, as that does not generate an externality. Using condition (14) then fully-funded clean technology support schemes cannot implement an efficient allocation with dirty output, $e_X > 0$.[1]

Remaining is the case with efficient pure clean production.

PROPOSITION 1 (ii). *Clean technology support with a fully-funded scheme uniquely implements an efficient allocation with pure clean production, $e_X = 0$, $e_Y > 0$.*

This result can be shown by using condition (14). The clean technology support schemes implement an efficient allocation characterized by pure clean production ($e_X = 0$), because an equilibrium attains condition (14) and the social efficiency condition $zB' = C'_Y$, when no environmental harm is created through production. With purely clean production, no environmental harm is created through production. This can occur with with no externality

[1] Notice that the sum of energy $e \equiv (e_X + e_Y)$ generated with dirty and clean technology is variable for reasons of plausibility. With fixed or minimum production $e > 0$ instead, fully-funded clean technology support implements optimal joint dirty and clean production ($e_X > 0$, $e_Y > 0$) as the quantity demanded no longer responds to the price for dirty and clean goods produced, that is, demand is price-inelastic. The planning problem reads choosing (e_X, e_Y) so to maximize $B(ze) - C_X(e_X) - C_Y(e_Y) - D(\psi e_X)$ subject to $e_X + e_Y \geq e$. At an interior solution, $C'_X + \psi D' = C'_Y$. The behavioral conditions and payment balance imply $(\varphi - C'_X)e_X + (\varphi - C'_Y)e_Y = 0$, which helps to express the optimal policy with taxes on the demand side as $\tau^C = \varphi - C'_X$ and $\sigma^C = \psi D'$ using the exogenous price for the dirty and clean good φ. Policy then implements an interior optimum by directing inputs without the need to scale them.

cost from dirty production at the margin. Moreover, policy uniquely implements an efficient allocation with pure clean production, hence no inefficient allocation results from policy that implements an efficient allocation.

Equivalency of Support Schemes in Terms of Welfare.—What can be said about the welfare that can be attained with fully-funded clean technology support schemes? By Proposition 1(i) and (ii), with efficient dirty production, the best allocation that can be achieved is constrained efficient, and else the welfare maximum can be attained. I will now show that for maximizing welfare constrained or unconstrained by full funding, any support scheme can be used, by establishing an equivalency result of these support schemes.

To establish the equivalency of support schemes, I will use a primal formulation of an optimal tax problem. Using this formulation, I will show that a constrained welfare-maximizing policy in a given scheme can be found using resource and implementability constraints, as in standard fiscal and monetary policy, as every clean technology support scheme representing a policy and price system supports certain allocations.

LEMMA 2. *(i) An allocation $x = (e_X, e_Y, q)$ in a competitive equilibrium with clean technology support satisfies the resource constraint (1) and the implementability constraint (14). (ii) Given an allocation that satisfies (1) and (14), one can construct a policy and a price system that together with the allocation constitute a competitive equilibrium.*

Notice that the implementability constraint (14) holds with joint dirty and clean production ($e_X > 0$ and $e_Y > 0$) and pure clean and dirty production ($e_X = 0$ and $e_Y = 0$), and the implementability constraint (14) uses that consumption is interior, $q > 0$.

The result in Lemma 2 helps to characterize welfare-maximizing policy π through choosing the allocation subject to (1) and (14). To show this, I will now define a Ramsey equilibrium so that a welfare-maximizing allocation appears as a Ramsey equilibrium allocation rule evaluated at a Ramsey equilibrium policy. A Ramsey equilibrium policy maximizes welfare.

A *Ramsey equilibrium* for clean technology support with tax/subsidy, quota, or standard is a policy π, allocation rules $q(\cdot)$, $e_X(\cdot)$, $e_Y(\cdot)$, and a price function $\phi(\cdot)$ on the set of admissable policies Π such that: The policy maximizes welfare, that is,

$$\tilde{\pi} = \arg\max W(q(\tilde{\pi}), e_X(\tilde{\pi}), e_Y(\tilde{\pi}))$$

subject to the payment-balance condition for type t, and every policy $\tilde{\pi}$, allocation $q(\tilde{\pi})$, $e_X(\tilde{\pi})$, and $e_Y(\tilde{\pi})$, and price system $\phi(\tilde{\pi})$ are a competitive equilibrium.

Before stating the equivalency result, notice that, as a result of Lemma 2, a welfare-maximizing allocation that a Ramsey equilibrium policy implements can be found by solving the problem of choosing the allocation (q, e_X, e_Y) so to maximize welfare $W(q, e_X, e_Y)$ subject to the resource constraint (1) and implementability constraint (14), and the side constraints $e_X \geq 0$ and $e_Y \geq 0$.

PROPOSITION 2. *The Ramsey equilibrium policies of clean technology support with tax, quota, or standard yield the same constrained efficient or efficient welfare level.*

The result in Proposition 2 covers both the cases of efficient joint dirty and clean production ($e_X, e_Y > 0$) and pure clean production ($e_X = 0 < e_Y$). The clean technology support schemes of subsidy/tax, quota, and standard yield the same constrained efficient welfare with both dirty and clean production, because these schemes have the same implementability constraint which can be stated as (14). Including $e_X \geq 0$ ($e_Y \geq 0$) as a constraint takes care of a Ramsey equilibrium allocation with no dirty (no clean) production. The schemes yield the efficient welfare level, hence reach the same welfare level, when no dirty production characterizes efficiency.

In numerical examples, I will use the general equilibrium setting described next.

2.H General Equilibrium

Households' preferences for consumption q, leisure $(1-L)$, and environmental quality Q are expressed through the utility function $U(q, 1-L, Q)$, twice continuously differentiable, increasing and strictly concave in the first and second, and increasing and concave in the third argument. To guarantee positive consumption and leisure, denoting partial differential with subscript, I assume the Inada conditions $\lim_{q \to 0} U_1 = \infty$ and $\lim_{\ell \to 1} U_2 = \infty$. Each household can spend one unit of time on labor supply in the market L or leisure at home $(1-L)$.

The consumption good can be produced with two technologies using labor

L_X and L_Y and using total factor productivity $z > 0$ according to

$$q = z(e_X + e_Y), \quad e_X = F_X(L_X), \quad e_Y = F_Y(L_Y). \tag{15}$$

Diminishing marginal returns in the production functions $F_X(\cdot)$ and $F_Y(\cdot)$ can represent the geographical variation of productivity in generating useful energy with dirty and clean technology. All labor is used in production,

$$L_X + L_Y = L. \tag{16}$$

As a regularity, production requires an input, $F_j(0) = 0$, $j \in \{X, Y\}$. One can think of labor producing capital, which in turn generates energy to align the model with the fact that energy production in practice currently is more capital-intensive than the average economy.

The environmental quality $Q = 1 - \psi E$ is reduced below a background level of 1 by dirty production with the exposure of $\psi > 0$,

$$E = F_X(\cdot).$$

For example, in the climate problem dirty energy creates carbon dioxide emissions that reduce environmental quality. In another problem nuclear energy creates a disutility while renewable energy does not.

3 Best Support Scheme and Best Emissions Tax

This section studies the best support scheme and the best emissions tax.

3.A Inelastic Labor Supply

The policy goal are reductions in emissions. The best support scheme yields lower emissions than in status quo, and, more importantly, higher welfare than the status quo.

The optimum can be implemented with the best support scheme only if pure clean production is optimal. This allows to rank welfare between the best support scheme and the best emissions tax rate: the emissions tax yields weakly higher welfare, and the same welfare only if pure clean production is optimal.

best support scheme yields higher/lower emissions for small/large exposure compared to the optimum, or the optimal emissions tax rate,

results unaffected by an income tax rate

3.B Elastic Labor Supply

I consider the two policies of a user tax $(\tau_E, 0)$, and the best fully-funded support scheme (α_X, α_Y) subject to (12), each fixing τ_L.

User Tax

To build intuition, I will write the decision problems of households and firms without normalizing prices and then normalize the wage rate.

Each identical household chooses (q, L) so as to maximize utility subject to the budget constraint

$$pq = (1-\tau)wL + \Pi + T,$$

taking as given the prices (p, w), profit Π, the lump-sum transfer T, and environmental quality $1 - \psi F_X$.

Each identical firm chooses (L_X, L_Y, u) to maximize the after-tax profit $\Pi = (1-\tau)[pG(u) - (\varphi + \tau^C)u + \varphi F_X(L_X) + (\varphi + \sigma^C)F_Y(L_Y) - w(L_X + L_Y)]$, taking as given the prices (p, φ, w), the tax rates (τ^C, τ), and the subsidy rate σ^C. Payment balance, expressing that the tax $(\tau^C(F_X + F_Y))$ equals the subsidy $(\sigma^C F_Y)$, implies (3). Demand equals supply on the market for the intermediate good, $u = F_X + F_Y$. Using the payment balance and market clearance, the budget constraint of the government holds in the form $\tau p G(u) = T$.

Consumer and producer behavior predict

$$\begin{aligned} &(1-\tau)(B' - \tau^C) \leq C'_X, \text{ with equality if } e_X > 0, \\ &(1-\tau)(B' - \tau^C + \sigma^C) \leq C'_Y, \text{ with equality if } e_Y > 0. \end{aligned} \tag{17}$$

This uses that firms equate the price of the final and intermediate good, $p = \varphi$, households set the marginal rate of substitution between consumption and leisure equal to their relative after-tax price, $U_2/U_1 = (1-\tau)w/p$, and the wage rate can be normalized to the inverse of the income tax factor, $(1-\tau)w = 1$. Equivalently, the price of the consumption good is normalized to U_1/U_2.

Producer Tax, Feebate, Tradeable Portfolio and Intensity Quota

Each identical firm chooses (L_X, L_Y) to maximize the after-tax profit $\Pi = (1-\tau)[(p - \alpha_X(\cdot))F_X(L_X) + (p - \alpha_Y(\cdot))F_Y(L_Y) - w(L_X + L_Y)]$, taking as given the prices (p, w), and the tax rates $(\alpha_X(\cdot), \alpha_Y(\cdot), \tau)$. Payment balance, expressing that the tax $(\alpha_X F_X)$ equals the subsidy $(-\alpha_Y F_Y)$, implies (12).

Using the payment balance, the budget constraint of the government holds in the form $\tau p G(u) = T$. Notice that I have consolidated the monetary cost of buying and the revenue of selling the intermediate goods.

Consumer and producer behavior predict

$$\begin{aligned} (1-\tau)(B' - \alpha_X) &\leq C'_X, \text{ with equality if } e_X > 0, \\ (1-\tau)(B' - \alpha_Y) &\leq C'_Y, \text{ with equality if } e_Y > 0. \end{aligned} \tag{18}$$

Payment balance and behavior (3) and (17), or (7) and (18), imply that

$$(B' - C'_X)e_X + (B' - C'_Y)e_Y = \tau B'[e_X + e_Y]. \tag{19}$$

3.C Pollution Relative to the Laissez-Faire and Optimum

Does Policy Lower Pollution Relative to the Laissez-Faire?—As clean technology support aims at lowering pollution, I will now examine whether the support schemes do so.

As a benchmark, consider the laissez-faire equilibrium allocation arising with policy variables set to zero. A scheme can be said to lower pollution if pollution decreases with an increase in the subsidy σ^C or $(-\alpha_Y)$, for example, from the value of zero. Under regular conditions on production, clean technology support schemes lower pollution, or emissions.

PROPOSITION 3. *With one of the production functions F_X or F_Y being strictly concave, an increase in the subsidy σ implementing interior allocations ($L_X > 0$, $L_Y > 0$) implies that:*

(i) Clean production e_Y increases.
(ii) Dirty production e_X decreases.
(iii) Emissions E decrease.

Overall production may increase when the subsidy increases. But emissions decrease as dirty production decreases.

As the production functions F_X and F_Y are concave, the marginal cost functions c_X and c_Y weakly increase in their output. With strictly concave F_X or F_Y (strictly increasing c_X or c_Y), the subsidy σ can be varied implementing different interior allocations. An increase in the subsidy can only be consistent with smaller L_X and greater L_Y, using () and (). Hence, starting from an interior allocation with no policy (hence $c_X = c_Y$) or with policy (hence $c_X \leq c_Y$), an increase in the subsidy σ leads to a decrease in the dirty goods and an increase in the clean goods (lower e_X and higher e_Y). The decrease in the dirty goods means that environmental quality increases. Under the assumption of strict concavity, the clean technology support schemes thus are effective.

Does Policy Yield Lower Pollution than in the Optimum?—An answer may be best based on a numerical example. In the example I study, the constrained optimal equals the optimal level of dirty production for some parameter values. Suppose the constrained optimal equals the optimal level of dirty production e_X. It can be proven that e_Y is too large at the constrained optimum.

I will now describe clean and dirty production under the best fully-funded clean technology support relative to the optimal production in an example. Production under the best fully-funded clean technology support is constrained efficient when joint dirty and clean production are optimal. An interesting pattern of constrained efficient production emerges.

For small exposure ($0 < \psi < \tilde{\psi}^0$), fully-funded support of clean production raises dirty production above its optimal amount. This is shown in Figure 1A where a curve represents the constraint (14) that describes the allocations implementable with clean technology support, and contours indicate different utility levels. The efficient allocation sits at the top of the utility contour set. The constrained efficient allocation yielded by the best support scheme resides on the curve representing the constraint which gives implementable allocations. For medium exposure ($\tilde{\psi}^0 < \psi < \tilde{\psi}^1$), the fully-funded support diminishes dirty production below its optimal amount and dirty production occurs. This is depicted in Figure 1B. For high exposure ($\tilde{\psi}^1 < \psi < \psi^1$), the fully-funded support eliminates dirty production albeit joint dirty and clean production are optimal. For upper levels of the exposure ($\psi \geq \psi^1$), the fully-funded support optimally prevents dirty production as the support implements the optimum with pure clean production.

For small to high exposure ($\psi < \psi^1$), fully-funded support of clean production implies too large clean production relative to its optimal amount. This can be seen from Figures 1A and 1B. For upper levels of the exposure ($\psi \geq \psi^1$), the fully-funded support yields the optimal amount of clean production as the support implements the optimum with pure clean production. Notice that the contour set of utility levels varies with the level of exposure.

Thus, by example, clean technology support may yield higher pollution

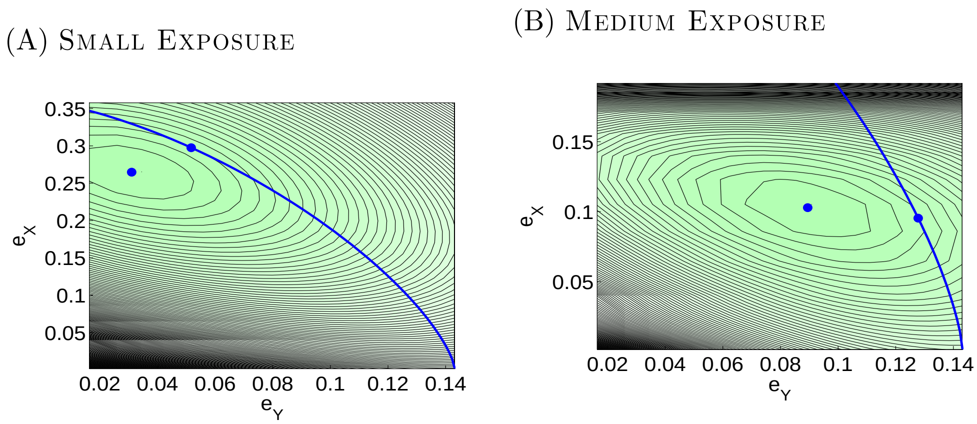


Fig 1. Efficient and Best Support Scheme Allocation: (A) Small Exposure; (B) Medium Exposure

when the pollution poses a weak problem and lower pollution when pollution poses a strong problem than at the optimum. The support may lift up clean production when the pollution poses a weak or strong problem relative to the optimum of joint dirty and clean production.

3.D The Allocations with Emissions Tax and Clean Technology Support

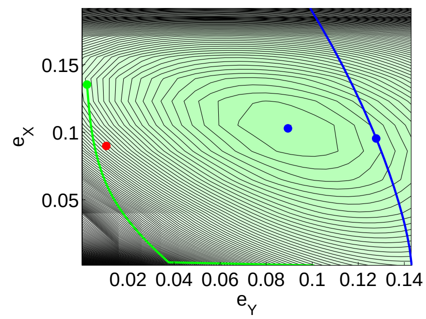
With τ_E, because emissions are proportional to dirty energy, the implementability constraint with an emissions tax follows as

$$(b - c_X)e_X = \tau b[e_X + e_Y].$$

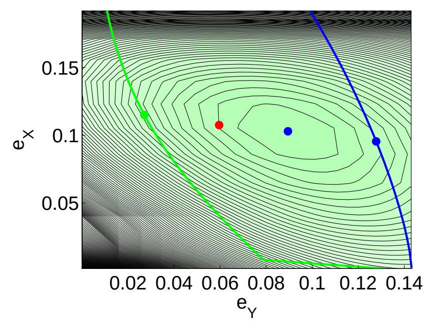
3.E Energy Conservation and Switching

Interpreting the intermediate good as energy, a dirty form of energy creates emissions while a clean form of energy does not create emissions. Reductions in emissions then can be achieved by conserving energy, that is, reductions in aggregate energy at given share of dirty energy, or by switching to clean energy, that is, increasing the share of clean energy at given aggregate energy use. This section examines differences in energy conservation and switching between the best support scheme and the optimum.

(A) High Income Tax Rate, $\tau = 0.87$



(B) Medium Income Tax Rate, $\tau = 0.60$



(C) Small Income Tax Rate, $\tau = 0.15$

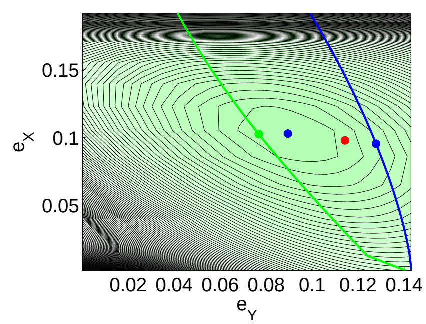


Fig 2. Efficient, Best Emissions Tax, and Best Support Scheme Allocation: (A) High τ (Income Tax Rate); (B) Medium τ; (C) Low τ

Decomposing Emissions Reductions.—Let us write the share of dirty energy in the status quo as $\eta_{X,0} = F_{X,0}/e_0$ ("old") and in an allocation implemented by the best support scheme or achieved at an optimum as $\eta_{X,1} = F_{X,1}/e_1$ ("new"). The old and new amount of aggegate energy are given as e_0 and

e_1. Then the difference in emissions can be written as

$$E_0 - E_1 = b(F_{X,0} - F_{X,1}) = b((F_{X,0}/e_0)e_0 - (F_{X,1}/e_1)e_1).$$

Defining energy conservation at the old distribution and energy switching at the new aggregate amount, the difference in emissions can be decomposed as

$$E_0 - E_1 = b[(\eta_{X,0})e_0 - (\eta_{X,0})e_1] + b[(\eta_{X,0})e_1 - (\eta_{X,1})e_1].$$

The Results.—The best fully-funded renewable energy support (RES) implies greater switching from dirty to clean energy and smaller conservation than optimal policy. The net effect on emissions reductions may, however, be smaller or greater for the best support scheme compared to the optimal policy.

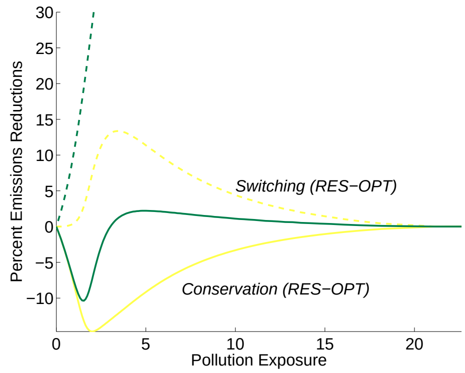
The difference in switching of emissions reductions relative to laissez-faire emissions follows an inversely U-shaped pattern dependent on the severity of pollution. This can be seen in Figure 3 (with emissions reductions in percent), as the curve for switching becomes bowed out for some medium level of exposure compared to low and high levels of exposure. Fully-funded renewable energy support induces less conservation of energy than optimal policy. The difference in conservation of emissions reductions relative to laissez-faire emissions follows a U-shaped pattern dependent on the severity of pollution. This can be seen in Figure 3 (with emissions reductions in percent), as the curve for conservation becomes bowed out for some medium level of exposure compared to low and high levels of exposure. Overall, the differences in energy switching and conservation with RES and optimal policy are largely offset and thus imply similar emissions reductions as depicted in Figure 3. For some exposure, fully-funded renewable energy support and optimal policy lead to the same emissions. For low exposure, the emissions are greater through RES, while for high exposure, the emissions are greater through optimal policy, confirming the results in Figure 1.[2]

[2] Another decomposition defines energy conservation at the new distribution and energy switching at the old aggregate amount,

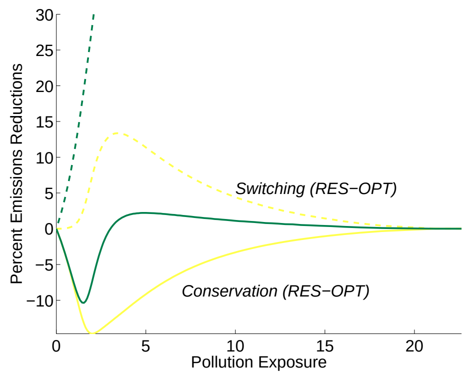
$$E_0 - E_1 = b((\eta_{X,1})e_0 - (\eta_{X,1})e_1 + (\eta_{X,0})e_0 - (\eta_{X,1})e_0).$$

The choice of the definition of conservation and switching (or weighting differently defined conservation and switching) does not matter for comparison between two allocations, such as the constrained optimal policy of renewable energy support and the optimum implemented by optimal clean and dirty goods taxes.

(A) High Income Tax Rate, $\tau = 0.87$



(B) Medium Income Tax Rate, $\tau = 0.60$



(C) Small Income Tax Rate, $\tau = 0.15$

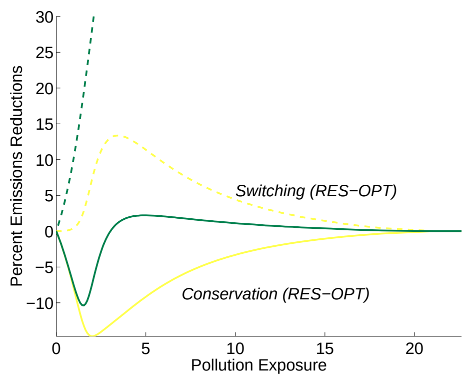


Fig 3. Conservation, Switching, and Net-of-Conservation-and-Switching Effect on Emissions Reductions: (A) High τ (Income Tax Rate); (B) Medium τ; (C) Low τ

4 Optimal Taxation with Clean Technology Support

In this section, I will show how fully-funded support schemes can be generally adjusted by a two-part tariff to achieve the optimum. In the case of elastic labor supply, when the income tax rate takes a unique level, the optimum can be implemented with a plain fully-funded support scheme.

To build intuition, suppose first that the income tax rate could be chosen. The policy that implements the optimum, including the general tax and clean good subsidy rate, follows as

$$\tau = (\eta_X/B')\tilde{\tau}, \tag{20}$$

$$\tau^C = (1-\eta_X)\left[\frac{1}{1-(\eta_X/B')\tilde{\tau}}\right]\tilde{\tau},$$

$$\sigma^C = \left[\frac{1}{1-(\eta_X/B')\tilde{\tau}}\right]\tilde{\tau}.$$

Optimal policy features the income tax rate given in (20) and the commodity tax rates given by

$$\alpha_X = \tau^C,$$

$$\alpha_Y = \frac{\eta_X}{1-\eta_X}(-\tau^C).$$

The profit tax rate that implements the optimum with fully-funded policy including a tax on the demand or supply side follows, by virtue of (3) or (12), directly from (19). Prices can be stated as $p = B'$ with taxes on the demand or supply side, and $\varphi = B'[1-(1/B')\tilde{\tau}]/[1-(\eta_X/B')\tilde{\tau}]$ smaller than B' with taxes on the demand side, and $\varphi = B'$ with taxes on the supply side. Fully-funded policies can then be constructed using the relationships given in Section 2.F.

The optimum can be implemented as three policy instruments affect two conditions reflecting behavior and one condition expressing payment balance characterizing fully-funded clean technology support. These conditions are to be evaluated at the optimum. To implement an optimum with joint dirty and clean production, we require a third instrument the general tax and specific subsidy rate, here given by the income tax rate.

Clearly, the income tax rate vanishes at an optimum with pure clean production. This confirms that fully-funded clean technology support with linear taxes and no income taxation implements an optimum with pure clean production (see Section 2.G).

To that I add the following example. To see the workings of taxation, consider a constrained efficient allocation that arises with no income taxation. At this allocation, $b < c_Y$ as $\tau = 0$. Suppose, in line with the example

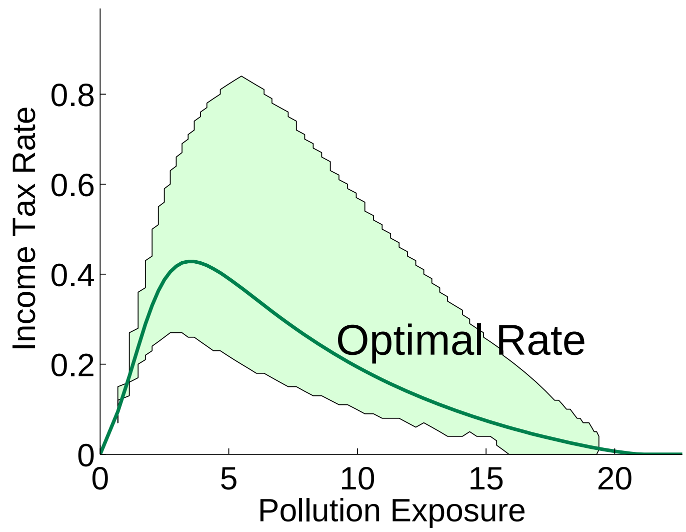


Fig 4. Ranking of Fully-Funded Schemes and Emissions Tax

presented in Section XXX, that the constrained efficient and efficient level of dirty production coincide. Then raising the income tax rate (τ) while keeping fixed dirty production (e_X) lowers clean production (e_Y) by increasing the marginal benefit b and possibly reducing the marginal cost of clean production c_Y—on the verge to equate them.[3]

4.A Ranking Policies with Preexisting Income Tax and Optimal Taxation

Figure 4 shows the ranking of fully-funded schemes and the policy of an emissions tax.

[3]Suppose on the contrary that clean production e_Y increases, causing b and c_Y to decrease and weakly increase. This can be contradicted as follows. The net revenue of dirty production $b-\tau^C$ or $b-\alpha_X(\cdot)$ depending on the fully-funded scheme moves in the same direction as the income tax rate τ, implying that the tax rate in the fully-funded scheme τ^C or $\alpha_X(\cdot)$ decreases, and the difference between the marginal cost of clean and dirty production, equal to $(1-\tau)\sigma^C$ or $(1-\tau)(-\alpha_Y(\cdot)$, weakly increases in clean production, implying that the subsidy rate σ^C or $-\alpha_Y(\cdot)$ increases. But that dirty relative to clean production decreases, using the payment balance (3) or (18), means that the tax relative to the subsidy rate increases, a contradiction. Hence, clean production decreases.

Nonlinear Tax. However, an optimum with negative externality—with pure externality-creating production or joint production ($e_X > 0 = e_Y$ or $e_X, e_Y > 0$), can be implemented with a nonlinear tax. A rising marginal tax schedule balances the linear subsidy, $-(1-\kappa)t-\kappa(t_X+t_Y) = W_X e_X > 0$. The amount overpaid by the linear subsidy equals the value of the missing market for a contribution to the public good by producing less of the externality-creating good ($W_X e_X$). In line with Proposition 1, at production with external cost, $e_X > 0$, efficiency requires that the marginal tariff for sellers decreases, $\psi(z) \geq \psi(z')$ for $z < z'$ and $\psi(z) > \psi(z')$ some $z < z' \leq \min\{e_X, e_Y\}$. Thus, the tax must increase.

Support of Tax. The support of optimal nonlinear tax policy can be reasoned as follows. The support of the constant marginal tax must include aggregate demand (m) if the subsidy is funded on the demand side ($\kappa < 1$), and must include the maximum of technology-specific supply, which equals demand ($e_Y = m$), if the subsidy is fully funded on the supply side ($\kappa = 1$). The tax can be steeper at higher quantities. The nonlinear tax must rise at least at the minimum of technology-specific supply (minimum of e_X and e_Y) if the tax is partially funded on the supply side ($\kappa \in (0, 1)$) to be effective. The tax can be flat for higher quantities.

Proposition 1 implies that:

Corollary 1. *A nonlinear tax that corrects a negative externality is progressive.*

Proposition 4. *(i)The policy π^B that implements the efficient allocation has the following properties:*

(i) In a two-part tariff, the lump-sum rebate (r) equals the marginal environmental cost times the amount of dirty output, $r = de_X$.
(ii) In a two-block tariff, the amount exempted from the marginal surcharge ($\tilde{x}$) equals the marginal environmental cost times the amount of output, $\tilde{x} = d(e_X + e_Y)$.
(iii) $\rho^{\mathrm{B}}(e) = 1$.
(iv) In any nonlinear tariff, the marginal surcharge φ can be constructed from $(\varphi - d)/\varphi = (e_X - \tilde{x})/e_X$ using d, e_X, and $\tilde{x}$ for $e_X > 0$.

subsidy to clean energy relative to the wholesale price σ/φ

Alternatively, efficiency can be implemented with a nonlinear tax buyers pay in addition to a base price. According to Proposition **??**, with externality, the tariff for buyers increases. Thus, the tax must increase with the quantity purchased. With prevented externality, the tariff is linear. Thus, the tax must be linear.

5 Summary and Outlook

Fully-funded clean technology support can be conducted in a variety of schemes—from general surcharge to specific funding and from intensity to percentage requirement of clean energy implying the same welfare with an environmental motive. Each scheme achieves the same constrained efficient or efficient welfare, because each scheme yields the same implementability constraint.

While the goal of reductions in dirty production can be achieved, the optimum is not implemented when some dirty production is optimal. The design of a fully-funded scheme with quantity-tagging such as through a two-part tariff helps to achieve efficiency. With elastic supply of factors such as labor there exists a tax rate on income from the supply of this implrmenting the optimum. With elastic labor supply, and the labor income tax rate deviating from this level, then a two-part tariff is needed to restore efficiency.

Future research may consider spillover effects which, as some argue, can form one rationale for renewable energy support schemes.

Future work may take the current approach to optimal clean energy support over to a multi-region world as the current analysis suggests that the optimal emissions tax differs over regions with different income tax rates.

Appendix A: Proofs

Proof of Lemma 1

The implementability constraints (I-1) and (I-2) require defining, for general surcharge, $\tau = p - \varphi$, for feebate, $\tau = \tau^*$, and for quota, $\tau = \alpha\upsilon$. The value for τ must be nonnegative as $e_X > 0$, $e_Y > 0$, and $\sigma \geq 0$. The value for τ must be smaller than b, as $e_X > 0$ requires that $c_X > 0$, so that $p - \tau > 0$. Use the first-order conditions of households [(i) in the definition of equilibrium] to obtain $p = b$. Clearly, then the first-order condition of producers with respect to L_X [part of (ii) in the definition of equilibrium] implies (I-1). Furthermore, the use of the first-order condition of producers with respect to L_Y [part of (ii) in the definition of equilibrium] can be stated as $p - \tau + \sigma = c_Y$. Substituting an expression for σ from the payment-balance condition for general surcharge, feebate, or quota (**??**), (6), or (**??**), in the form $\tau(e_X + e_Y) = \sigma e_Y$ then yields (I-2). QED

Proof of Lemma **??**. The result amounts to Samuelson's (1954) condition on the optimal provision of a privately produced public good for the special economy deployed here. With identical households and equal (unitary) welfare weights, a planner seeks to solve the problem

$$\text{With respect to } \; x_i \geq 0, \ell_i \in [0,1], \text{ all } i \in [0,1], L_X \geq 0, L_Y \geq 0, \lambda \geq 0, \epsilon \geq 0$$

$$\begin{aligned}\text{Max} \int_0^1 & U(x_i, 1 - \ell_i, 1 - w_X F_X(L_X))di \\ & + \lambda \left[F_X(L_X) + F_Y(L_Y) - \int_0^1 x_i di \right] \\ & + \epsilon \left[\int_0^1 \ell_i di - L_X - L_Y \right].\end{aligned}$$

An interior solution ($x_i > 0$ some i, $\ell_i \in (0,1)$ some i, still $L_X \geq 0$ and $L_Y \geq 0$) requires that $x_i = x$, $\ell_i = \ell$, all $i \in [0,1]$, and

$$\left[\frac{U_1}{U_2} - w_j \frac{U_{1-w}}{U_2} \right] (\partial F_j / \partial \ell_j) \leq 1, \quad \text{with equality if} \quad e_j > 0, \;\; j \in \{X, Y\},$$

with the marginal utility with respect to environmental quality and labor ($\partial U / \partial(1-w)$ and $\partial U / \partial(1-\ell)$) denoted as U_{1-w} and U_2. Here I have used that $e_j > 0$ if and only if $\ell_j > 0$. Using the definition of b, c_X, and c_Y in

Section **??**, and $\tilde{w}_X$ and $\tilde{w}_Y$ in Section **??**, then yields the desired result. QED

Proof of Proposition 1(i)

By Lemma **??**, efficiency with $e_X > 0$ requires that $(b - d_X)/c_X = 1$. Then condition (I-1) shows that $\tau > 0$ in equilibrium with an externality cost from dirty technology, $d_X > 0$. Furthermore, $\eta_X/(1-\eta_X) > 0$ if $e_X > 0$ and $e_Y > 0$. But condition (I-2) then implies that in equilibrium $b < c_Y$. By Lemma **??**, efficiency with $e_Y > 0$ requires that $b = c_Y$, contradicting $b < c_Y$. This yields the desired result. QED

Proof of Proposition 1(ii)

As a preliminary step, I need to show that condition (14) holds, which the next Lemma accomplishes.

LEMMA A.1. *An allocation X with $L_X = 0$ in a competitive equilibrium with general surcharge, feebate, or quota satisfies the implementability constraint (14).*

Proof. For $L_X = 0$, condition (14) holds when $p = c_Y$ if $e_X = 0$. That $e_X = 0$ requires that $e_Y > 0$, because $\ell \in (0,1)$. Producer optimality with $L_Y > 0$ for a general surcharge requires that $\varphi + \sigma = c_Y$, for a feebate implies that $p + \sigma^* = c_Y$, and for a quota shows that $p + \upsilon = c_Y$. Inserting the expression for the unit revenue into the corresponding payments-balance condition (**??**), (6), and (**??**), where $e_X = 0$, then yields the desired result. QED

In step 1, I show that given an allocation $x > 0$, $\ell \in (0,1)$, $L_X = 0$, and $L_Y > 0$, the resource constraints (1) and (16), and the implementability constraint (14), there exist a policy and price system that, together with the allocation, constitute a competitive equilibrium. (This step I will use in the proof of Proposition 2 as well). In step 2, I show efficiency. In step 3, I establish that given this policy, there exists no competitive equilibrium characterized by $L_X > 0$.

Step 1(a). This substep confirms household and producer optimality, and prepares finding profits and the budget constraint of households. I consider each scheme separately. *General Tax.* Let $p = b$, $\varphi = 0$, and $\sigma = c_Y$. The relative price of consumption and leisure (p) satisfies household optimality [part of (i) in the definition of equilibrium]. The price and policy φ and σ clearly satisfy producer optimality with respect to L_X and L_Y [part of (ii) in the definition of equilibrium] with $L_X > 0$ and $L_Y > 0$. Notice that $0 \leq c_X$, as c_X is finite or zero. Furthermore, $b = c_Y$ following from (14), and the resource constraint (1), $\int x_i di = e_Y$, yield that $p \int x_i di = c_Y e_Y$. To obtain the payment-balance condition, expand the right side of this expression using the policy and price system, $p \int x_i di = \varphi e_X + (\varphi + \sigma) e_Y$, where $e_X = 0$ and $e_Y > 0$ represent aggregate dirty and clean goods produced. *Feebate.* Let $p = b$, $\tau^* = b$, and $\sigma^* = 0$. The remainder follows analogously to the general surcharge. The payment-balance condition becomes $p \int x_i di = (p - \tau^*) e_X + (p + \sigma^*) e_Y$. *Quota I.* Let $p = b$, $\upsilon = 0$, and $\alpha = \infty$ with $\alpha\upsilon = c_Y$. The remainder follows analogously to the general surcharge. The payment-balance condition becomes $p \int x_i di = (p - \alpha\upsilon) e_X + (p + \upsilon) e_Y$.

Step 1(b). Assume profit Π by using the resource constraint (16), $\int \ell_i di = L_X + L_Y$, and writing $p \int x_i di = \Pi + \int \ell_i di$ [completing (ii) in the definition of equilibrium]. The profit for every producer is identical. Then the aggregate budget constraint of households, $\Pi = px - \ell$, results, which also appears to be the individual budget constraint of identical households [completing (i) in the definition of equilibrium].

Step 2. Lemma **??** shows necessary conditions for efficiency. These conditions presumably hold with $L_X = 0$ and $L_Y > 0$, so that $b = c_Y$. That condition and the resource constraints are also sufficient for a solution to the planner's problem with $L_X = 0$ and $L_Y > 0$. The given policy for each scheme, allocation, and price system constituting a competitive equilibrium yield these conditions. Thus, the policies attain efficiency.

Step 3. Notice that $p = b > 0$ in any equilibrium. The remainder is a proof by contradition. Also here I consider each scheme separately. *General Tax.* Suppose that $e_X > 0$ with the policy $\sigma = c_Y$. Dirty output exceeds zero only if $\varphi = c_X > 0$. But then $(\varphi + \sigma) > c_Y$, contradicting that in equilibrium $(\varphi + \sigma) \leq c_Y$. *Feebate.* Suppose that $e_X > 0$ with the policy $\tau^* = b$ and $\sigma^* = 0$. Dirty output exceeds zero only if $p - \tau^* = c_X > 0$. But then $p - \tau^* + \sigma^* = c_X$, so that (14), for $e_Y > 0$ and $e_Y = 0$, implies that $px = (p - b)x$, contradicting that in equilibrium $p = b > 0$. *Quota I.* Suppose that $e_X > 0$ with the policy $\alpha = \infty$. As $e_X > 0$ implies that $e_Y / e_X < \infty$, then $\alpha = e_Y / e_X$ could not be enforced. QED

Proof of Proposition 2

$\underline{L_X = 0 < L_Y}$

Part i. The resource constraints (1) and (16) hold in a competitive equilibrium by definition [part of (iii) in the definition of equilibrium]. For $L_X = 0$, condition (14) follows from the preliminary step in the proof of Proposition 1(ii).

Part ii. Step 1 in the proof of Proposition 1(ii) shows the desired result.

$\underline{L_X, L_Y > 0}$

Part i. The resource constraints (1) and (16) hold by definition of equilibrium. For $L_X, L_Y > 0$, condition (14) follows from combining the conditions (I-1) and (I-2).

Part ii. As in the proof of Proposition 1(ii), use two parts (a) and (b).

Step 1(a). Specific to the clean technology support scheme, show the first-order conditions of households and producers, and prepare finding profit and the budget constraint of households. *General Tax.* Let $p = b$, $\varphi = c_X$, and $\sigma = c_Y - c_X$. Setting the price $p = b$ satisfies household optimality [(i) in the definition of equilibrium]. The price and policy φ and σ satisfy producer optimality with respect to L_X and L_Y [part of (ii) in the definition of equilibrium] with $L_X > 0$ and $L_Y > 0$. Direct use of the condition (14) with the resource constraint (1), $\int x_i di = e_X + e_Y$, reveals the payment-balance condition (**??**), $\int x_i di = \varphi e_X + (\varphi + \sigma) e_Y$, where $e_X > 0$ and $e_Y > 0$ represent aggregate dirty and clean goods produced. *Feebate.* Let $p = b$, $\tau^* = b - c_X$, and $\sigma^* = c_Y - b$. The remainder follows analogously to the general surcharge. The payment-balance condition becomes $p \int x_i di = (p - \tau^*) e_X + (p + \sigma^*) e_Y$. *Quota I.* Let $p = b$, $\upsilon = c_Y - b$, and $\alpha = e_X / e_Y$. The remainder follows analogously to the general surcharge. The payment-balance condition becomes $p \int x_i di = (p - \alpha\upsilon) e_X + (p + \upsilon) e_Y$.

Step 1(b). The part (b) of Step 1 in the proof of Proposition 1(ii) shows the desired result.

$\underline{L_X > 0 = L_Y}$

Part i. An allocation with $L_Y = 0$ in a competitive equilibrium satisfies the resource constraints (1) and (16) by definition of equilibrium. The next Lemma shows that the implementability constraint (14) holds.

LEMMA A.2. *An allocation X with $L_Y = 0$ in a competitive equilibrium with general surcharge, feebate, or quota satisfies the implementability constraint (14).*

Proof. For $L_Y = 0$, condition (14) holds when $p = c_X$ if $e_Y = 0$. That

$e_Y = 0$ requires that $e_X > 0$, because $\ell \in (0,1)$. Producer optimality with $L_X > 0$ for a general surcharge implies that $\varphi = c_X$, for a feebate requires that $p - \tau^* = c_X$, and for a quota shows that $p - \alpha\upsilon = c_X$. Inserting the expression for the unit revenue into the corresponding payment-balance condition (**??**), (6), and (**??**), where $e_Y = 0$, then yields the desired result. QED

Part ii. The case $L_Y = 0$ considered here can arise in equilibrium with virtually no policy (general surcharge policy specified as $\sigma = 0$, feebate policy $(\tau^*, \sigma^*) = (0,0)$, or quota policy $\alpha = 0$). The prices are $p = b$ (with general surcharge, feebate, or quota) and $\varphi = b$ with general surcharge. Using similar arguments as for the cases with $L_Y > 0$ above shows the household and producer optimality, the profit function, and the budget constraint of the households. QED

Ramsey Equilibrium

The payment-balance condition for a type of clean technology support can be stated as one of (**??**), (6), and (**??**). That is, for general surcharge, $p(\tilde{\sigma})x(\tilde{\sigma}) = \varphi(\tilde{\sigma})F_X(\cdot) + (\varphi(\tilde{\sigma})) + \tilde{\sigma})F_Y(\cdot)$. For feebate, $\tilde{\tau}^* F_X(\cdot) = \tilde{\sigma}^* F_Y(\cdot)$. For quota, $p(\tilde{\alpha})x(\tilde{\alpha}) = (p(\tilde{\alpha}) - \tilde{\alpha}\upsilon(\tilde{\alpha}))F_X(\cdot) + (p(\tilde{\alpha}) + \upsilon(\tilde{\alpha}))F_Y(\cdot)$.

Proof of Proposition 2

By Lemma 2, a welfare-maximizing allocation that a Ramsey equilibrium policy implements maximizes utility, that is, solves the problem $\max_{x,\ell,L_X,L_Y} U(\cdot)$, subject to the resource constraints (1) and (16), the implementability constraint (14), and the side constraints $L_X \geq 0$ and $L_Y \geq 0$. Notice that the case of joint use of dirty and clean technology not only arises in a welfare-maximizing allocation. It can arise in the laissez-faire equilibrium, for virtually no policy, with sufficiently diminishing returns to scale in producing the clean good. QED

Proof of Proposition 3

Proof: The proof works by contradiction. Case 1. Suppose that L_X and L_Y increase. Then x would increase, because ℓ increased, as indicated by the resource constraints () and ().Notice that $b = U_1/U_2$ decreases in x and ℓ as utility increases in x and $(1-\ell)$ with strict concavity in x and $(1-\ell)$. Then

b would decrease, $b^1 < b^0$, denoting old and new values by superscript 0 and 1. Moreover, ${c_X}^1 \geq {c_X}^0$ and ${c_Y}^1 \geq {c_Y}^0$. The changes in the marginal benefit and marginal cost contradict () resulting from () and (). Case 2. L_X and L_Y decrease. Then x and ℓ would decrease, which together with the direction of changes in the marginal costs, analogously to the Case 1, contradicts () resulting from () and (). Case 3. L_X increases and L_Y decreases. Then ${c_X}^1 \geq {c_X}^0$ and ${c_Y}^1 \leq {c_Y}^0$ violate the increase in the subsidy which implies that $\sigma^1 = {c_Y}^1 - {c_X}^1 > {c_Y}^0 - {c_X}^0 = \sigma^0$ resulting from (). QED

Appendix B: Further Quotas and Standards

TO DO.

References

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