Revisiting the EEG: Dynamic Equilibrium with Rational Expectations on the Housing Market

Marc Baudry[1,2] and Nathaly Cruz[*,1,2,3]

Economix, UPL, Univ Paris Nanterre, CNRS, F92000 Nanterre France
Climate Economics Chair, F75002 Paris France
Centre Scientifique et Technique du Bâtiment, F77447 Champs-sur-Marne France

Abstract. This paper proposes a Dynamic theoretical approach in order to revisit the *Energy Efficiency Gap* in the housing market. By contrast with the previous literature, the explicit dynamic setting allows a rigorous analysis of the role of expectations and of their interplay with equilibrium in the housing market. In this paper we show that, in a pure competition context and with a continuum of landlords having heterogeneous discount rates, if landlords have perfect expectations some of them refrain from investing in energy efficiency because they know that energy efficient housing will become more abundant in the future and therefore capitalization of energy efficiency will decrease. We also propose an extension of the model to a stochastic dynamic context where the price of energy may fluctuate as time goes. This extension establishes a bridge between the core of the model based on hedonic price modeling and the strand of literature that builds on real option theory to deal with the *Energy Efficiency Gap*. Implications for the measurement of the expected capitalization of energy efficiency based on the estimation of hedonic price functions are discussed.

Key words: Hedonic prices, Energy Efficiency Gap, dynamic theoretical approach, housing market, expectations

1 Introduction

Nowadays energy transition policies are encountering numerous challenges. One of the most pressing and important challenges deals with the transition to more efficient and less intensive modes of energy consumption of buildings (Lovins, 1998; Ademe, 2017) . Indeed, in most OECD's countries, buildings are making use of an important amount of energy that contributes to fossil fuel depletion and to the emission of greenhouse gases and, as a consequence, to global warming. In spite of public policies that have been implemented in order to boost the adoption of energy efficient solutions and curve energy consumption, a persistent divergence is observed between targeted efficiency and consumption and what is actually observed. This difference is known as the Energy Efficiency Gap -EEG (Hirst and Brown, 1990; Allcott and Greenstone, 2012). Since it has been identified in the earliest 90's, the EEG has been documented for a large number of cases, including but not limited to the building sector. For Hirst and

Brow (1994) "the energy efficiency gap is the unexploited economic potential for energy efficiency (...). It emphasizes the technically feasible energy efficiency measures that are cost-effective but are not being deployed". Likewise, Jaffe and Stavins (1990) refer to the unexploited advantages of current performing technologies and propose several types of explanations to the phenomenon of EEG[1]. Understanding the causes of the EEG is crucial for the design of efficient public policies that help getting around the barriers to the adoption of energy efficient solutions and switch to lower energy consumption. This paper argues that, at least in the specific case of the building sector, it is probably required to go beyond the usual explanations and to switch to a dynamic partial equilibrium approach of the real estate market if one is intended to fix the EEG.

In a broad classification of explanations proposed by the academic literature for the EEG, a distinction is usually made between behavioral explanations and market failure explanations. The behavioural approach to the EEG consists in explaining "why observed behaviour is indeed optimal from the point of view of energy users" (Jaffe and Stavins, 1994). A large number of studies in this strand of literature rely on a multidisciplinary approach that tries to combine, for instance, social psychology and economics. Bounded rationality, prospect theory or the concept of heuristic decision making are, among others, mobilised to explain deviation of observed behaviors from behaviors predicted by the standard decision theory prevalent in microeconomic analysis (see e.g. Crosby, 2006). In this vein of literature, the attitude towards risk is also a natural candidate to explain why a supposedly cost-effective solution does not meet success in the real world. Dietz (2010) more specifically investigates how risk perception affects the evaluation peoples make of savings induced by energy efficient devices. When risk affects the dynamics of a component of the return from investing in energy efficient solutions and this investment induces a sunk cost, the real option theory popularised by Dixit and Pindyck (1994) suggests that, even if they are risk neutral, investors will more easily postpone the investment compared to a standard Net Present Value decision criteria. As a theory of "conservative" behaviors, the real option theory is thus able to explain part of the EEG. The papers by Kumbaroğlu *et al.* (2012) or Lee (2014) are illustrative of a category of works that highlights how more conservative behaviors as regards investments in energy efficiency result from the real option decision criteria compared to the net present value criteria.

The market failure approach to the EEG rather put the emphasis on the economic context surrounding the investment decision without questioning the rationality of investors. Information asymmetries are typically a source of divergence between observed and predicted behaviors. Myers (2015), for instance, empirically explores how asymmetries of information between landlords and tenants in the northwest of the United States induce such a divergence. Another related market failure discussed by Davis (2011) is the misalignment of economic

[1] For a complete state of the art, see Gerarden et al. (2017) https://www.aeaweb.org/articles?id=10.1257/jel.20161360 and Gerarden et al. 2014 https://www.aeaweb.org/articles?id=10.1257/aer.p20151012

incentives between landlords and tenants. As long as the cost of inefficient technologies is beared by tenants and is often subject to information asymmetries, landlords have no strong incentives to invest in more efficient technologies. In the spirit of the seminal work of Stiglitz and Weiss (1981) on credit rationing, limited access to capital may also explain under-investment in energy efficiency for buildings.

All these different kinds of explanation of the EEG gap share a common feature: they all explicitly or implicitly rely on the assumption that the energy efficiency solutions studied are profitable in the sense that investing in these solutions yields a positive net present value as long as the investor is risk neutral and there is no sunk cost of investment. This paper breaks with these previous works by considering that even the standard NPV criteria for risk neutral investors in the absence of sunk cost is misleading to assess the profitability of energy efficient solutions. Put another way, this paper questions the concept of cost-effectiveness in the definition of the EEG paradox given *supra*. The key idea is that previous works overestimate the return on investment by adopting a decision theory approach rather than a partial equilibrium approach. Indeed, they analyse the decision of a single economic agent as if this agent was the only one facing to opportunity to invest. This is generally a modeling choice, but it may also be thought of as the outcome of naive expectations of investors as regards the functioning of the real estate market. By contrast, this paper emphasizes the polar case of investors having perfect foresight or rational expectations of the dynamics of the real estate market in pure competition. It makes use of the concept of hedonic price equilibrium proposed by Rosen (1974), extended to a dynamic setting. More precisely, it shows that if investors are aware that many buildings may be subject to upgrading in terms of energy efficiency they rationally anticipate a lower capitalisation of energy efficiency at market equilibrium in the future.

Section 2 displays the main modeling assumptions and provides a brief adaptation of Rosen's concept of hedonic price equilibrium to the context of the paper. Section 3 presents the basic model. In a two periods and two types of buildings context, investment decision in energy efficiency by heterogeneous landlords under different expectation schemes is examined. The emphasis is more specifically on the two polar cases of naive *versus* rational expectations. Section 4 extends the analysis to the context of a stochastic dynamics of energy prices. It provides an original real option modeling of investment decisions when the dynamics of returns on investment are partly endogenous due to the adjustment of equilibrium hedonic prices. Section 5 concludes with a discussion of the empirical implications for the measurement of the capitalisation of energy efficiency in real estate prices and its use as a correct indicator of the return of investment in energy efficiency.

2 Model Setting

2.1 Modeling Assumptions

The housing market considered in the model consists in a rental market where, at each period, tenants choose the housing that maximizes their utility. Housings are vertically differentiated according to the subutility level H that they generate from their intrinsic characteristics (that resumes to the living space in meter square in the detailed modeling proposed in Appendix A) and from a heating temperature T. The cost of heating per unit of temperature is $c_j = v/k_j$ with v the unit price of energy and k_j the level of capital embedded in the housing j that contributes to energy efficiency of that housing. The higher k_j, the lower the cost c_j of heating for a given price of energy v. The income of a tenant i is denoted by R_i and the price of a housing providing a subutility level H_j is P_j. To keep the model computationally tractable, the model follows Shaked and Sutton (1982) and relies on a total utility for each tenant i that is linear:

$$U_{ij} = \gamma_i * H_j + X \tag{1}$$

where X stands for the aggregate level of goods, other than housing services, that is purchased by i. This aggregate of "other goods" is used as the *numeraire*. γ_i is a preference parameter specific to i. Following the lines detailed in Appendix A, after substitution of the budget constraint $X = R_i - P_j - c_jT$ for tenant i if it is located in a housing of type j in the utility function and rearrangement, the utility level obtained by tenant i conditionally on location in housing j may be written as

$$U_{ij} = \alpha_i * H_j + R_i - P_j \tag{2}$$

Where α_i is a term specific to tenant i and represents her preferences for energy quality. The exact expression of H_j and α_i depends on additional assumptions on the temperature T discussed in the next paragraph. The model distinguishes between three types of housing.

The first two types gather all the housings owned by private landlords, with one different landlord for each housing so that the market for housings is in pure competition on the supply side. The first type, referred to by subscript B, corresponds to low energy efficiency houses. Conversely, the second type referred to by subscript A corresponds to high energy efficiency houses. To keep things simple, all other characteristics of houses are assumed to be identical between types A and B. As a result, a same subutility level can be associated to each type and is denoted respectively H_A and H_B for types A and B. Appendix A details the computation of this subutility level when tenants have Cobb Douglas preferences on the combinations of heating temperature and other characteristics of housing. Appendix A highlights that, whether a rebound effect in the consumption of energy is taken into account or not, the subutility level always depends on the unit price v of energy with $H_B < H_A$ whatever this unit price. The scenario with no rebound effect is captured by taking the heating temperature as exogenous so that the expenditure level in heating is just the product of the exogenous heating temperature T and the unit cost of temperature $c_j = v/k_j$. It is shown in Appendix A that H is then proportional to v with a positive coefficient of proportionality that directly reflects how much capital k has been invested for the housing under consideration in energy efficient solutions. Moreover, the difference $H_A - H_B$ in subutility levels increases in v. By contrast, in the scenario with a rebound effect the tenants optimally choose the heating temperature conditionally on k_j. As heating is cheaper for high energy efficient housings, tenants tend to choose a higher heating temperature in these housings. The increase in heating temperature has an opposite effect compared to energy efficiency in terms of energy consumption, which is consistent with the rebound effect as defined in the literature (see e.g. Greening, 2000; Hens, 2010). In Appendix A, this leads to a decrease of each subutility level H in v whatever the level of energy efficiency measure by k and to a decrease of the difference $H_A - H_B$ in v. Henceforth, we focus on the scenario without a rebound effect in order to better identify the effects of price adjustments on the market for housings.

The third type of housings considered in the model is social housing. These housings are property of public authorities which set their price P_S endogenously. Tenants have to satisfy predefined criteria to be able to ask for a social housing[2]. It is assumed that there is an excess of demand for social housing in the sense that there are more tenants eligible for social housings than units of social housings available. Without loss of generality, it is assumed that the subutility level H_S generated by social housings is the lowest of the three types of housings and this may result either from a lower value of k_S or a lower level of other characteristics (e.g. a lower living space). Although our focus is on investment decision by private landlords, introducing social housing in the model has two purposes. First, an "outside the market" choice is required to determine the equilibrium prices in a hedonic price model. Second, instead of assuming an unrealistic "outside the market" alternative with $H = 0$ and $P = 0$, social housing is the most realistic alternative in many developed countries. There are N_1 tenants eligible for social housings and N_2 tenants that are not eligible for social housings. Following the vertical differentiation model of Shaked and Sutton (1982), heterogeneity among tenants is captured by assuming that the preference parameter $\alpha_i > 0$ is drawn from a distribution with a cumulative distribution function $F_1(\alpha)$ if the tenant is eligible for social housing and a cumulative distribution function $F_2(\alpha)$ if the tenant is not eligible for social housing. Thereafter, wherever a specification of these distributions is required, a uniform continuous distribution on the intervals $[\alpha_{1min}, \alpha_{1max}]$ and $[\alpha_{2min}, \alpha_{2max}]$ is used for respectively $F_1(\alpha)$ and $F_2(\alpha)$.

Heterogeneity on the supply side of the model affects the discount rate (ρ), used by landlords to discount the flow of cost and revenues associated to the housing they own. The discount rate for a landlord l is supposed to be drawn from a distribution with a cumulative distribution function $G(\rho)$. Wherever a specification of this distribution is required, a uniform continuous distribution on the interval $[\rho_{min}, \rho_{max}]$ is used. Such a heterogeneity may capture differ-

[2] the criteria may be, for instance, a maximum level for income

ences among landlords in terms of access to capital and to the credit market. As discounting is a key element in the decision to invest or not in energy efficiency, heterogeneity in the discount rate is expected to explain why all landlords who could invest in energy efficiency do not systematically do it. The choice of the discount rate to capture heterogeneity in the model is also motivated by a tendency in the literature dealing with the EEG paradox to consider that discount rates have to be adjusted to fix the paradox (Jaffe and Stavins, 1994; Gerarden et al. 2017). As will be outlined latter in the paper, most of our discussion of the EEG relies on a comparison of threshold values for the discount rate. The distribution of discount rates is not conditional on the type of housing that landlords own. It is also considered that type A corresponds of housings corresponds to highest energy efficiency feasible given the state of technology. Said another way, the model deals with the adoption decision of innovative energy efficient solutions for buildings that are available, not with the generation of these innovative solutions. Consequently, landlords owning a type A housing face no investment opportunities. Only landlords owning a type B housing face an opportunity to upgrade it to a type A in counterpart of an sunk investment cost I which is assumed time invariant and identical for all landlords and housings. In order to keep the model computationally tractable, it is assumed that is no time to build when upgrading a housing.

2.2 Hedonic Price Equilibrium

Prior developing the dynamic model in interest is the next sections, it is worthwhile highlighting that the basis of the modeling approach builds on the discrete version of the original model proposed by Rosen (1974). More precisely, it is a version of Rosen's model where there is group-wise heterogeneity in the supply side[3]. This in done in three steps. In a first step, the location decision of tenants is discussed. In a second step, a single type of housings proposed by private landlords is considered and the equilibrium price for this type is obtained. In a third step, the presentation is extended to two types of housings proposed by private landlords. In the last two steps the analysis is static (only one time period) and the supply is assumed to be sticky in the sense that no investment opportunities is considered for the time being. Nevertheless, results will be useful for the two periods dynamic analysis discussed latter, even with flexible supply.

A tenant i characterized by a preference parameter α_i prefers a type j housing rather than a type l if and only if it provides him a higher utility level. In this case we have

$$\alpha_i H_j + R_i - P_j > \alpha_i H_l + R_i - P_l \tag{3}$$

Assuming that $H_j > H_e$, this yields the following condition on the preference parameter α:

$$\alpha_i > \frac{P_j - P_l}{H_j - H_l} \tag{4}$$

[3] By contrast, Baudry and Maslanskaia-Pautrel (2016) analyze a version of Rosen's model with group-wise heterogeneity on the demand side

Thus, if $J+1$ types of housing ranked in the increasing order of the corresponding subutility levels H_j ($j = 0, ..., J$ and $j = 0$ corresponds to $j = S$), the population of tenants will be partitioned in J groups according to their preference parameter α so that tenants with a α_i in the interval $[\frac{P_j - P_{j-1}}{H_j - H_{j-1}}, \frac{P_{j+1} - P_j}{H_{j+1} - H_j}]$ will choose a type j ($j = 1, ..., J-1$) housing and tenants with a α_i above the threshold $\frac{P_J}{H_J}$ will chose a type J housing.

If only types S and A of housings are available on the market, tenants that are not eligible to social housings will necessarily locate on a type A housing[4]. Tenants that are eligible to social housing have to choose between type S and type A. Given that there are N_1 tenants eligible to social housing and N_2 tenants that are not eligible, the total number of tenants locating on type S and type A housings are respectively given by

$$\Theta_S^{Demand} = N_1 F_1(\frac{P_A - P_S}{H_A - H_S}) \tag{5}$$

$$\Theta_A^{Demand} = N_1(1 - F_1(\frac{P_A - P_S}{H_A - H_S})) + N_2 \tag{6}$$

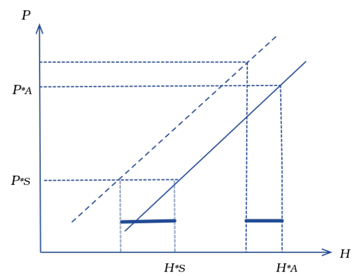
If Θ_S^{Supply} and Θ_A^{Supply} respectively stand for the number of social housings and housings of type A available on the supply side of the market, then market equilibrium in the spirit of Rosen (1974) requires that $\Theta_S^{Demand} = \Theta_S^{Supply}$. Note that this condition necessarily implies that $\Theta_A^{Demand} = \Theta_A^{Supply}$. Solving this equilibrium condition, the equilibrium price for types A of housings is

$$P_A^* = P_S + (H_A - H_S) F_1^{-1}(\frac{\Theta_S^{supply}}{N_1}) \tag{7}$$

where $F_1^{-1}(.)$ is the inverse cumulative distribution of the preference parameters α of tenants that are eligible to social housings. This equilibrium price outlines the key role played by the "outside the market" alternative. Figure 1 illustrates this equilibrium. The continuous line that goes through the two points H_S, P_S and H_A, P_A correspond to the bid function of tenants that are eligible to social housings and are just indifferent between type S and type A. The bid function of a tenant i with preference parameter α_i is the maximum amount of money E_i the tenant is willing to pay to switch from a combination $\{H_j, P_j\}$ to an alternative combination $\{H, E_i\}$. It solves the indifference relation between the two combinations and is given by $E_i = P_j + \alpha_i(H - H_j)$. In space (H, P) its slope is thus α_i. Consequently, the slope of the line on Figure 1 is $\frac{P_A - P_S}{H_A - H_S}$. The set of points $\{\{H_S, P_S\}, \{H_A, P_A\}\}$ in Figure 1 visualize the hedonic price relation in this basic modeling. Contrary to Rosen's model this relation is not visualized by a continuous curve but by a set of discrete points because of group-wise heterogeneity on the supply side. It has been outline *supra* that in the case of a

[4] Anticipating on the backward resolution of the two periods dynamic model, it makes sense to consider that at the second period all privately owned housings will be of type B.

positive shock on the price of energy and in the absence of a rebound effect, the difference $H_A - H_S$ increases. As P_S is exogenous, it follows on that $P_A^* - P_S$ increases. Figure 1 also illustrates this change. The dotted line is the bid curve of the new pivot tenants that are indifferent between the two alternatives at this new equilibrium.



4 Figure1

Fig. 1. Static equilibrium with a sticky supply when only types S and A of housings are available

If both types A and B of private housing are available on the market, the equilibrium conditions state that the demand for each type just equals the number of housing of this type available on the supply side. The demand for type S is unchanged compared to the previous case except that subscript B replaces subscript A (equation 8). The remaining tenants N_1- who have not chosen social housings are partitioned between types A and B according to the value of their preference parameter α. Similarly, the whole population N_2 of tenants that are not eligible to social housing are partitioned between types A and B. Consequently, the demands for each type of housings for enxogenously given prices are

$$\Theta_S^{Demand} \equiv N_1 F_1(\frac{P_B - P_S}{H_B - H_S}) \tag{8}$$

$$\Theta_B^{Demand} = N_1(F_1(\frac{P_A - P_B}{H_A - H_B}) - F_1(\frac{P_B - P_S}{H_B - H_S})) + N_2 F_2(\frac{P_A - P_B}{H_A - H_B}) \tag{9}$$

$$\Theta_A^{Demand} = N_1(1 - F_1(\frac{P_A - P_B}{H_A - H_B})) + N_2(1 - F_2(\frac{P_A - P_B}{H_A - H_B})) \tag{10}$$

On the supply side, Θ_S^{Supply} Θ_B^{Supply} and Θ_A^{Supply} stand respectively for the number of social housings, housings of type B and housings of type A available. Market equilibrium requires that each demand level equates the corresponding supply levels. Solving $\Theta_S^{Supply} = \Theta_S^{demand}$ yields the equilibrium price for types B:

$$P_B^* = P_S + (H_B - H_S) F_1^{-1}(\frac{\Theta_S^{Supply}}{N_1}) \tag{11}$$

The equilibrium price P_B^* may then be substituted in the equilibrium condition (9) for types B which yields

This condition implicitly defines the equilibrium price P_A^* of type A. As long as the distributions $F_1(.)$ and $F_2(.)$ are distinct (i.e. the distribution of the preference parameter α is not independent of being eligible to social housing or not) the explicit expression of P_A^* can not be obtained. In the specific case where $F_1(.)$ and $F_2(.)$ are identical (and denoted by $F(.)$) because the distribution of α is independent of being eligible to social housing or not, then the explicit expression of P_A^* can be obtained and reads

$$P_A^* = P_B^* + (H_A - H_B)F^{-1}\left(\frac{\Theta_S^{Supply} + \Theta_B^{Supply}}{N_1 + N_2}\right) \tag{12}$$

If, in addition, F is continuous and uniform on the interval $[\alpha_{min}, \alpha_{max}]$ then the last term in the right hand side of the previous expression is linear in $\Theta_S^{Supply} + \Theta_B^{Supply}$ and reads

$$F^{-1}\left(\frac{\Theta_S^{Supply} + \Theta_B^{Supply}}{N_1 + N_2}\right) = \alpha_{min} + \frac{\Theta_S^{Supply} + \Theta_B^{Supply}}{N_1 + N_2}(\alpha_{max} - \alpha_{min}) \tag{13}$$

Figure 2 illustrates the equilibrium. The continuous line going through points $\{H_S, P_S\}$ and $\{H_B, P_B^*\}$ corresponds to the bid curve of tenants that are just indifferent between the two corresponding alternatives and similarly for the continuous line going through points $\{H_B, P_B^*\}$ and $\{H_A, P_A^*\}$. The dotted lines show how the equilibrium is affected by an increase in the price of energy. Such an increase implies a drop of sub-utility levels H_S H_B and H_A. In the absence of a rebound effect, the differences $\{H_B - H_S\}$ and $\{H_A - H_B\}$ are wider than the initial ones so that the pivot tenants that are just indifferent between the different alternatives change and the equilibrium prices increase.

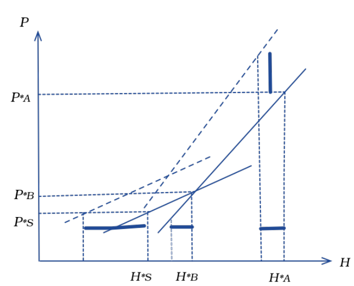


Fig. 2. Static equilibrium

Fig. 3. Static equilibrium with a sticky supply when types S, A and B of housing are available

3 Basic Model: Net Present Value

3.1 Decision Theory Solution Based on NPV Criterion: Not Possibility of Reporting

A common practice when dealing with the *Energy Efficiency Gap* consists in assessing whether a energy efficiency solution is profitable or not on the basis of the Net Present Value criteria. For this purpose, the expected capitalization of a retrofit (i.e. switching from a type B to a type A housing) can serve as a measurement of the return on investment. A landlord with bounded rationality nevertheless observes the past capitalization. Let Θ_A^{IN} and Θ_B^{IN} denote the numbers of housings of types A and B respectively just before the current period referred to as $t = 0$. Let $P_{A_0}^{IN}$ and $P_{B_0}^{IN}$ denote the associated static prices. The landlord is aware that these prices are sensitive to the price of energy because the impact of variations of this price on market equilibrium has been observed in the past. It is thus assumed that the landlord is able to correctly assess this impact and thus use $E_0[P_j^{IN}]$ ($j = A$ or B) for future values of P_j where E_0 stands for the expected value conditional on the knowledge of the current value v_0 of the price of energy and its stochastic process. The values of $P_{A_1}^{IN}$ and $P_{B_1}^{IN}$ conditional on the price v_1 of energy at date $t = 1$ are obtained as the outcome of the static hedonic price model discussed in Subsection 2.2. Therefore, in order to decide whether to invest or not at $t = 0$, a Landlords with bounded rationality and a discount rate ρ will use the following NPV criteria

$$V_{NPV}^{isolated} = Max \begin{cases} P_{A_0}^{IN} - I + \dfrac{E_0[P_{A_1}^{IN}]}{(1+\rho)} & \text{if investment in t=0} \\ P_{B_0}^{IN} + \dfrac{E_0[P_{B_1}^{IN}]}{(1+\rho)} & \text{if no investment} \end{cases} \tag{14}$$

Investment occurs if and only if

$$\frac{E_0[P_{A_1}^{IN} - P_{B_1}^{IN}]}{(1+\rho)} > I - (P_{A_0}^{IN} - P_{B_0}^{IN}) \tag{15}$$

At this stage, an additional assumption is crucial to proceed. It is assumed that the increase $P_{A_0}^{IN} - P_{B_0}^{IN}$ in the rent following on from investment at $t = 0$ is not sufficient to counterbalance the sunk cost of investment I. Said another way, it is necessary to account for the gain at time $t = 1$ for investment to be profitable. If this assumption was not used, then the problem would not be a dynamic problem. Given this assumption, the right hand side in the previous condition is positive and the condition is equivalent to:

$$\rho < \rho_{NPV}^{isolated} \text{ with } \rho_{NPV}^{isolated} = \frac{E_0[P_{A_1}^{IN} - P_{B_1}^{IN}]}{I - (P_{A_0}^{IN} - P_{B_0}^{IN})} - 1 \tag{16}$$

Consequently, if all landlords had bounded rationality the proportion of housings of type B that would be upgraded to type A at $t = 0$ would be given by

$$\Delta_{NPV}^{isolated} = G(\rho_{NPV}^{isolated}) \tag{17}$$

The proportion $\Delta_{NPV}^{isolated}$ may be thought of as the degree of additional adoption of energy efficient solutions predicted by academic works that apply basic economic calculus. We now turn to the prediction made with a more subtle analysis that accounts for the fact that there is a continuum of landlords facing the opportunity to upgrade their housing.

3.2 Perfect Market Expectations Based on NPV Criterion: Not Possibility of Reporting

A rational landlord is aware that there is a continuum of landlords owning a type B of housing and considering the opportunity to upgrade it to a type A. Said another way, he is aware that there is flexibility on the supply side. If such a landlord makes rational expectations, then he applies the following Net Present Value criteria to decide whether or not to invest

$$V_{NPV}^{rational} = Max \begin{cases} P_A(v_0) - I + E_0[(P_A(v_1)] \dfrac{1}{(1+\rho)} & \text{if investment in t=0} \\ P_B(v_0) + \dfrac{E_0[(P_B(v_1)]}{(1+\rho)} & \text{if no investment} \end{cases} \tag{18}$$

The noticeable difference compared to the expression (16) of $V_{NPV}^{isolated}$ is that prices are no longer those computed with the values Θ_A^{IN} and Θ_B^{IN} of the numbers of housings of types A and B observed before $t = 0$ but those computed endogenously at equilibrium in $t = 0$ and denoted Θ_A^0 and Θ_B^0. As there is no additional expected investment in $t = 1$ if the NPV criteria is used, the equilibrium prices $P_B(v_1)$ and $P_B(v_1)$ for $t = 1$ are computed as in the static model detailed in Subsection 2.2 with Θ_A^0 and Θ_B^0 in place of respectively Θ_A^{IN} and Θ_B^{IN}. A rational landlord thus invest in $t = 0$ if and only if

$$P_A(v_0) - I + \frac{E_0[P_A(v_1)]}{(1+\rho)} > P_B(v_0) + \frac{E_0[P_B(v_1)]}{(1+\rho)} \tag{19}$$

which may be rearranged as :

$$\frac{E_0[P_A(v_1) - P_B(v_1)]}{(1+\rho)} > I - (P_A(v_0) - P_B(v_0)) \tag{20}$$

As for the switch from (16) to (17), it is assumed that the increase $P_A(v_0) - P_B(v_0)$ in the rent following on from investment at $t = 0$ is not sufficient to counterbalance the sunk cost I so that it is necessary to account for the gain at time $t = 1$ for investment to be profitable. Then, the right hand side in the previous condition is positive and the condition is equivalent to

$$\rho < \rho_{NPV}^{rational} \text{ with } \rho_{NPV}^{rational} = \frac{E_0[P_A(v_1) - P_B(v_1)]}{I - (P_A^0(v_0) - P_B^0(v_0))} - 1 \tag{21}$$

so

$$\rho_{NPV}^{rational} < \rho_{NPV}^{isolated} \tag{22}$$

4 Extended Model: Option Value

4.1 Decision Theory Criterion with Possibility of Reporting

A limit to the analysis done so far is that the opportunity to invest and upgrade a housing of type B to a housing of type A is considered only at time $t = 0$ and the opportunity to do it at time $t = 1$ is not taken into account. In order to correctly deal with the possibility to delay the investment rather than forgive it, a real option modeling is required. A well known result of real option theory is that more conservative decisions emerges. Nevertheless, real option models seldom deal with the simultaneous decisions of multiple economic agents facing the same kind of opportunity and the resulting interplay between these decisions.

As a first step toward the analysis of the interplay between the decisions of the multiple landlords in the model, let reconsider the decision of a landlord with bounded rationality in a real option decision criteria in place of a net present value criteria. To keep things tractable, let consider that only two events are possible as regards the dynamics of the price of energy: v_1 takes either value $v_1^{sup} > v_0$ with probability z or value $v_1^{inf} < v_0$ with probability $(1 - z)$. Moreover, for the option problem to make sense, it is assumed that investing at time $t = 1$ if not already done at time $t = 0$ is optimal only in the case of a positive shock on the price of energy (i.e. $P_A^{IN}(v_1^{sup}) - P_B^{IN}(v_1^{sup}) < I$ but $P_A^{IN}(v_1^{inf}) - P_B^{IN}(v_1^{inf}) > I$). Then the option value decision criteria corresponds to the following problem.

$$V_{OV}^{isolated} = Max \begin{cases} P_A^{IN}(v_0) - I + \dfrac{E_0[P_A^{IN}\tilde{v_1})]}{1+\rho} \\ \text{if investment in } t = 0 \\ \\ P_B^{IN}(v_0) + \dfrac{E_0[P_B\tilde{v_1}) + z[P_A^{IN}(v_1^{sup}) - P_B^{IN}(v_1^{sup})] - I}{(1+\rho)} \\ \text{if the decision is postpone to } t = 1 \end{cases} \tag{23}$$

if the decision is postpone to t=1

Investment at t=0 is optimal iff:

$$\frac{E_0[P_A(\tilde{v_1})] - E_0[P_B(\tilde{v_1})] - z(P_A^{IN}(v_1^{sup}) - P_B^{IN}(v_1^{sup}) - I)}{(1+\rho)} \tag{24}$$

is higher than

$$I - (P_A^{IN}(v_0) - P_B^{IN}(v_0)) \tag{25}$$

Then

$$\rho < \frac{E_0[P_A^{IN}(\tilde{v_1}) - P_B^{IN}(\tilde{v_1})]}{I - (P_A^{IN}(v_0) - P_B^{IN}(v_0))} - 1 - z\frac{P_A^{IN}(v_1^{sup}) - P_B^{IN}(v_1^{sup})}{I - (P_A^{IN}(v_0) - P_A^{IN}(v_0))} \tag{26}$$

with $\rho_{OV}^{isolated} < \rho_{NPV}^{isolated}$ because postponement is always more profitable than definitive abandonment and where the prices $P_B^{IN}(v)$ and $P_B^{IN}(v)$ are those computed in the static model.

4.2 Perfect Market Expectations with Possibility of Reporting

In this model, there are two likely effects on v, v_{sup} for the positive one and v_{inf} for the negative one (Demonstration on Appendix D). Individuals take account of the energy shocks and the market adjustments through the Θ.

If the shock is positive all landlords invest in t=1 (and the effect on $(H_A - H_B)$ is proportional to the energy price (v) variation when there is not a possible rebound effect at it has been demonstrated on Appendix B). On the contrary, If the shock in negative: There is not any investment on t=1

For these two cases and following the flow of the Appendix D, we assume that:

$$v^{sup} > v_{lim1} \tag{27}$$

[5]

and

$$v^{inf} < v_{lim2} \tag{28}$$

[6]

In this model, the shocks on the energy price can generate two situations that lead to $(P_A(v^{sup})$, $(P_B(v^{sup})$, $(P_A(v^{inf})$ and $(P_B(v^{inf})$. These prices for two models, the first one with a whole rate of landlords that invest on energy quality for the negative shock (Equilibrium with (Θ_A+Θ_s=1) et an other with any investment (sticky offer equilibrium without uncertainty about v). If landlords are assumed to make inter-temporal choices and the presence of multiple investors their decision problem corresponds to the following option value problem:

$$Max \begin{cases} P_A(v_0) - I + E[(P_A(v^1)]\dfrac{1}{(1+\rho)} & \text{if invest in t=0} \\ P_B(v_0) + \dfrac{z[P_A(v_1^{sup}) - I] + (1-z)[P_B(v_1^{Inf}]}{1+\rho} & \end{cases} \tag{29}$$

Then investment occurs iff:

$$\rho < \rho_{OV}^{multiple} \text{ with } \rho_{OV}^{multiple} = \frac{(1-z)(P_A^*(v_1^{inf}) - P_B^*(v_1^{inf})) + zI}{I - (P_A^*(v_0) - P_B^*(v_0))} \tag{30}$$

And there are two possible cases (1) and (2).

(1) If prices do not change we obtain that

$$P_A^*(v_1^{inf}) - P_B^*(v_1^{inf}) = P_A^{IN}(v_1^{sup}) - P_B^{IN}(v_1^{sup}) \tag{31}$$

and

$$P_A^*(v_0) - P_B^*(v_0) = P_A^{IN}(v_0) - P_B^{IN}(v_0) \tag{32}$$

2. If prices change, it is necessary to increase the P^* in order to maintain the equilibrium, so θ_B decrease and θ_A increase

[5] the value of v_{lim1} is the same that we calculated on the case where all landlords invest

[6] the value of v_{lim2} is the same that we calculated on the case where there is not investments

$$P_A^* - P_B^* = (H_A - H_B)F^{-1}\left(\frac{\Theta_B^{Supply} + \Theta_S^{Supply}}{N_1 + N_2}\right) \tag{33}$$

for the NPV case we had that

$$P_A^*(v_1^{inf}) - P_B^*(v_1^{inf}) < P_A^{IN}(v_1^{sup}) - P_B^{IN}(v_1^{sup}) \tag{34}$$

and

$$P_A^*(v_0) - P_B^*(v_0) < P_A^{IN}(v_0) - P_B^{IN}(v_0) \tag{35}$$

By comparing the equations (16) and (30), we deduce that:

$$\rho_{OV}^{multiple} < \rho_{NPV}^{isolated} \tag{36}$$

5 Conclusions

This paper has focused on the different making decision processes and on the dynamics of a the energy efficiency market when Landlords can invest to improve of the energy quality of the houses. This work takes place on two polar cases: naive and rational expectations about the dynamics of the real estate market. We include dynamics on three first models trough the evolution of energy prices and on the fourth one trough the global market adjustments. Based on the results presented on sections 3 and 4, we pustule that rate of houses that will passe from H_B to H_A, where individuals can postpone the decision of investment or when their improve their capacity to predict future, tend to be lower than equilibrium based on more naive expectations and that do not take account of the market adjustments. We demonstrate partially that the cost-effectiveness of investments is superior when individual do not anticipate the future value of the energy quality. We find two main instances than can explain the over-evaluation of the cost-effectiveness of investments on energy efficiency quality on the NPV models: 1. The fact of neglecting on the empirical works, the individuals trend to wait when the option to attend is possible (demonstrated on section 4) and 2. the fact that a high expectation of prices could generate the opposite effect when individuals place their decision in relation to the whole market. We aim to widen this inequalities observed in terms of investments while differed heuristics of rationally are developed on the empirical step of this research. This paper point out also, the remarkable necessity to take into account the complexity of real agents' refurbishment decisions. These decisions made on an inter-temporal scenario are determined by the heterogeneity of preferences for the present, two phenomenons usually discussed and described by the literature but rarely integrated on modeling.

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A Complementary Model of Tenants' Utility Function

In this complementary model, we explicit the Tenant's Utility function that depends of the level of heating chosen. We also demonstrate the existence of a rebound effect when Tenants can choose the level of temperature or their houses.

B Tenant's Utility Function

We suppose that tenants locate on the different houses according to their heterogeneous preferences for the energy quality (expressed by the parameter $\alpha_i > 0$. Individuals dispose of a income denoted R and pay a price energy P_j (that correspond to the rent before rental fees for heating) to a housing of quality j. The energy price affects simultaneously the costs of heating trough the parameter c. We also assume a real state with M houses of equal surface (S). The houses disposes of a technical energy performance indicator, K_j that ca be improved by refurbishment works funded by Landlords. We consider a homogeneous source of energy and the costs depends exclusively of the level of temperature T and the exogenous price of energy, v.H_j represents the level of energy quality of housing. Individuals also dispose of a parameter of preferences for heating β_i.

$$\beta, \in [0,1] \tag{37}$$

$$c = v\ k_j \tag{38}$$

As the housing surface is fixed, H_j indicate a sub-utility function for the energy quality service provided by the housing.

$$H = S^{\beta} T^{1-\beta} \tag{39}$$

Tenant's utility depends of energy quality multiplied by a parameter of preferences for energy quality γi, and the consumption of all others good X.

$$X = R - P - cT \tag{40}$$

$$U_i = \gamma_i H_j + (R - P - cT) \tag{41}$$

As the temperature is assumed as exogenous an exogenous parameter τ, we exclude in a first time the possibility of a rebound effect:

$$U_i = \gamma_i H_j + (R - P - c\tau) \tag{42}$$

$$U_i = \gamma_i (S^{\beta} \tau^{1-\beta}) + (R - P - c\tau) \tag{43}$$

at the optimum:

$$U_i^* = \gamma_i (S^{\beta} \tau^{1-\beta}) - \frac{v}{k} + R - P) \tag{44}$$

As $S_A = S_B$ so

$$\gamma_i (S^{\beta} \tau^{1-\beta}) \tag{45}$$

(45) is supposed identical for the housing A and B.

On the other hand, we know that:

$$-\frac{v}{k_A} + R - P_B > -\frac{v}{k_B} + R - P_A \tag{46}$$

and

$$k_A > k_B \tag{47}$$

so

$$H_A > H_B \tag{48}$$

and

$$\varepsilon_{H_A - v} = 1 \tag{49}$$

From (49) we conclude that without rebound effect, the elasticity of H to v is proportional and positive to v.

$$v\left(\frac{1}{k_B} - \frac{1}{k_A}\right) > 0 \tag{50}$$

B.1 Rebound Effect

By introducing the opportunity to choose T, when v changes.

The optimal conditions for T:

$$T* = c^{-1/\beta} S((1-\beta)\gamma_i)^{\frac{1}{\beta}} \tag{51}$$

And by substituting on the utility function:

$$U_i^* = \gamma_i^{\frac{1}{\beta}}[(1-\beta)^{(\frac{1}{\beta})-1}\beta]c^{1-(\frac{1}{\beta})S}(S^\beta T^{*1-\beta}) + R - P \tag{52}$$

And

$$\gamma_i^{1/\beta}[(1-\beta)^{(1/\beta)-1}\beta] \tag{53}$$

(53) is the term associated to the individual preferences for energy quality housing. We name this term α_i.

By comparing the optimal situation with the utility function, we deduct that:

$$H_j = c^{1-(1/\beta)} \tag{54}$$

This term is independent of individual preferences and ca be written as follow:

$$H_j = \frac{v}{k}^{1-(1/\beta)} \tag{55}$$

We deduce that the elasticity H to v is:

$$\varepsilon_{Hjtov} = 1 - \frac{1}{\beta} \tag{56}$$

$$\varepsilon_{Hjtov} < 0 \tag{57}$$

The elasticity does not depend of the individual preferences but of the parameter β

We know that for a steady β:

– c increase on v, then H decrease on v.
– c decrease on k, then H increase on k.

Energy Price Elasticity of the Demand for Energy Quality

For an increasing of energy prices, the reduction of H could be compensated by an increasing of K. For a constant energy performance of the houses K constant, individuals will be able to allocate a higher amount of R to a same energy quality service because of the rise of energy costs for heating.

$$H_j = \frac{v}{k}^{1-(1/\beta)} \tag{58}$$

$$H_j = v^{1-(1/\beta)} K_j^{(1/\beta)-1} \tag{59}$$

For two levels of energy quality: $j =_{A,B}$, we obtain:

$$\varepsilon_{H_A - v} = 1 - \frac{1}{\beta} \tag{60}$$

$$\varepsilon_{H_B - v} = 1 - \frac{1}{\beta} \tag{61}$$

Then the differential of prices can be written as:

$$H_A - H_B = v^{-(1-\beta)/\beta} k_A^{(1-\beta)/\beta} k_B^{(1-\beta)/\beta} \tag{62}$$

$$\frac{1}{H_A - H_B} = V^{(1-\beta/\beta)} \frac{1}{k_A^{(1-\beta)/\beta} k_B^{(1-\beta)/\beta}} \tag{63}$$

$$H = \varphi(c) \tag{64}$$

As

$$H_j = \varphi\left[\frac{v}{k_j}\right] \tag{65}$$

As φ is a constant and negative elasticity, when v decrease c decrease and the differential $(H_A - H_B)$ increase.

It was attended that the demand for high energy quality the demand for a higher energy quality increase with and increase of v. Nevertheless, H_j increase in v and by the equation () we observe an negative effect on the energy quality sous-utility. Individuals compensate the reduction of the costs of heating with an increase of the level of heating. That phenomenon could be interpreted as the rebound effect, identified empirically by a large number of research studies (Berkhout et al. 2000 ; Greening et al. 2000; Hens et al. 2010). The rebound effect can be readily identified on the equation of optimal temperature for the optimization program of the tenant.

$$T* = ((v)/k)^{-1/\beta} S((1-\beta)\gamma_i)^{1/\beta} \tag{66}$$

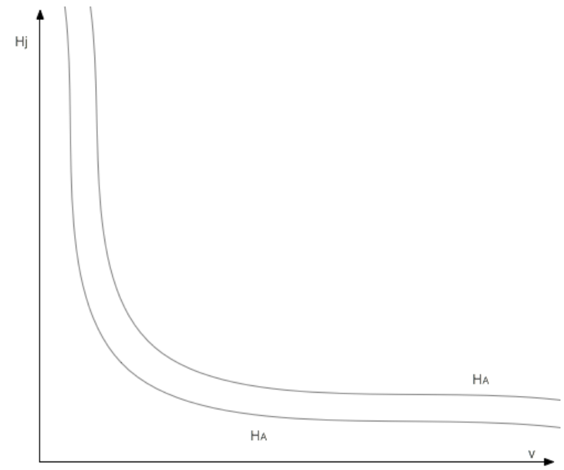

Fig. 4. Sub-utility energy quality function and energy price

B.2 Energy Effects of Energy Shock on the Market Equilibrium when Individuals Do Not Take Account of the Evolution of Energy Prices

In this section we suppose a perfect rationality of individuals. Prices do affect the equilibrium as far as the evolution of exogenous shocks on energy prices modify the level on energy quality preferred for each tenant on the equilibrium state by landlords do not take account of this phenomenon. We suppose that the effects on energy quality prices P_A and P_B on the second period do not modify the market equilibrium but the shock on energy prices generate changing on the demand side and not on the offer side. Without any profit perspective Landlords that owns a low level of quality housing do not have any incentive to invest on the improvement on energy quality different to incentives found for the case of hedonic equilibrium. In this case, the system takes account of the variation of the energy prices but investors do not. In t=0, the expected equilibrium is the same of in the hedonic prices case but the real equilibrium on the second period not.

E[P_A^1 - P_B^1] is the same that E[$P_A^{rational}$ - $P_B^{rational}$] The effect on the energy prices is the same that in the rational case but the value of energy quality differential is lower and will generate for the hole system equilibrium a lower number of houses of type A and a higher number of houses type B. The System Equilibrium is, in this case, calculated by using the same system of equations than in the hedonic prices.

For Tenants we obtain:

$$E_0\left[y\left(H_A - H_B\right)\right] = E_0\left[P_A - P_B\right] \tag{67}$$

$$yE_0\left[\left(H_A - H_B\right)\right] = E_0\left[P_A - P_B\right] \tag{68}$$

For the dynamical model, we use a countdown process. By calculating the future price of the high quality level houses in a first time, we determine the rate of houses and the investments on improvements of the energy quality level in the period t=1.

$$yE_0\left[\left(V^{\frac{\beta-1}{\beta}}\right)\left(K_A^{\frac{1-\beta}{\beta}}-K_B^{\frac{1-\beta}{\beta}}\right)\right]=E_0\left[P_A-P_B\right] \tag{69}$$

$$E_0\left[P_A-P_B\right]=yE_0\left[\left(V^{\frac{\beta-1}{\beta}}\right)*\left(K_A^{\frac{1-\beta}{\beta}}K_B^{\frac{1-\beta}{\beta}}\right)\right] \tag{70}$$

cause

$$H=\left(\frac{v}{k}\right)^{1-\frac{1}{\beta}} \tag{71}$$

$$H=c^{1-\frac{1}{\beta}} \tag{72}$$

$$c=\frac{v}{k} \tag{73}$$

$$\left(P_A-P_B\right)=\Omega_1\left(H_A-H_B\right) \tag{74}$$

$$E_0\left(P_A-P_B\right)=\Omega_1E_0\left(H_A-H_B\right) \tag{75}$$

And as

$$\left(H_A-H_B\right)=\left(\frac{v}{k_A}-\frac{v}{k_B}\right)^{1-\frac{1}{\beta}} \tag{76}$$

$$\left(H_A-H_B\right)=\left[\frac{1}{v}\left(k_A-k_B\right)\right]^{1-\frac{1}{\beta}} \tag{77}$$

$$\left(H_A-H_B\right)=v^{\frac{\beta-1}{\beta}}\left[k_A^{\frac{1-\beta}{\beta}}k_B^{\frac{1-\beta}{\beta}}\right] \tag{78}$$

Then

$$E_0\left(P_A-P_B\right)=\Omega_1E_0\left[v^{\frac{\beta-1}{\beta}}\left(k_A^{\frac{1-\beta}{\beta}}-k_B^{\frac{1-\beta}{\beta}}\right)\right] \tag{79}$$

$$E_0\left(P_A-P_B\right)=\Omega_1\left[k_A^{\frac{1-\beta}{\beta}}-k_B^{\frac{1-\beta}{\beta}}\right]E_0\left[v^{\frac{\beta-1}{\beta}}\right] \tag{80}$$

$$E_0\left(P_A-P_B\right)=\Omega_1\left[k_A^{\frac{1-\beta}{\beta}}-k_B^{\frac{1-\beta}{\beta}}\right]\left[zv_{sup}^{\frac{\beta-1}{\beta}}+(1-z)\,v_{inf}^{\frac{\beta-1}{\beta}}\right] \tag{81}$$

B.3 Effects on the Investment of Positive and Negative Shocks on Energy Prices

For the model with sticky supply for the second period, we have:

$$E[P_A(v_0)-P_B(v_O)]=\frac{D_1D_2M+N_1D_2\alpha_1+N2D_1\alpha_2-D_1D_2\Theta_A^{IN}}{D_2N_1+D_1N_2}(H_A-H_B) \tag{82}$$

or

$(\mathrm{P}_A-P_B)=(H_A-H_B)\eta$
(83)

Where η is a coefficient that depends of the characteristics of the distributions, N_1, N_2 and Θ_s.

We know also that $(H_A - H_B)$ is proportional to the energy price (v) cause there is not a rebound effect (Appendix A)

thus:

$$(P_A - P_B) > I \tag{84}$$

and

$$(H_A - H_B)\eta > I \tag{85}$$

By the expression of $(H_A - H_B)$ (in appendix A) we obtain:

$$(\frac{1}{k_B} - \frac{1}{k_A})v\eta > I \tag{86}$$

and

$$v > \frac{I}{(\frac{1}{k_B} - \frac{1}{k_A})\eta} \tag{87}$$