Monopoly, unilateral climate policies and limit pricing*

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Abstract

We examine the behavior of a fossil fuel monopolist who faces demand from two regions: a 'climate club' and the 'rest of the world' (ROW). Each region is able to produce a perfect substitute for fossil energy at constant marginal costs. The climate club uses a carbon tax and a renewables subsidy as policy instruments. The ROW is policy-inactive. We show that, due to differences in climate policies between the climate club and the ROW, the monopolistic fossil fuel supplier may choose for two limit-pricing phases to postpone entry of renewables producers: First in the clime club and later in the ROW. As soon as energy demand from the climate club shifts from fossil fuels to renewables, the monopolist abruptly *increases* the fossil price for the ROW. If the monopolist starts with limit-pricing in the climate club from the beginning, a renewables subsidy and a carbon tax lead to an *increase* in current resource use. If the initial oil price is below the limit price, however, both policy instruments cause a *decrease* in current resource use. The renewables subsidy speeds up resource depletion, whereas the effect of the carbon tax on the depletion time is ambiguous.

JEL codes: Q31, Q37

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1 Introduction

A broad distinction can be made between countries or regions that conduct an active climate policy and countries or regions that implement no policy at all or a lax policy. For example, the EU can be considered as being more concerned with climate than some African or Latin-American regions. There exists a large literature on the interaction of countries in this case. The phenomenon of carbon leakage as a consequence of unilateral policies is amply studied (cf. Hoel, 1991; Babiker, 2005; Burniaux and Martins, 2012; Van der Werf and Maria, 2012). However, two aspects are usually not taken into account. The first is that climate change is to a large degree caused by the use of fossil fuel from exhaustible resources (oil, gas and coal). The second is that the supply of such resources cannot be characterized as purely competitive. Eichner and Pethig (2011), Hoel (2011) and Ryszka and Withagen (2014) address exhaustibility and policy differences between regions under perfect competition, whereas Andrade de Sá and Daubanes (2016) and Van der Meijden and Withagen (2019) study the effects of climate policy under monopolistic resource supply, where fossil demand comes from a single region. In this paper we address these three issues—multiple regions, non-renewable resources, and monopolistic supply—in a unified framework to more closely examine the interaction between market structure and differences in policies between regions. In particular we want to answer the question how the market equilibrium looks like in a dynamic setting where two regions differ in the intensity of climate change policies and supply of fossil fuel comes from a monopoly. Consequently, after a full characterization of the equilibrium we want to investigate the impact of more strict environmental policies. For example, will a tightening of climate policies in one countries lead to faster overall supply and consumption of fossil fuel?

Our approach is closely related to earlier work by Van der Meijden et al. (2018). The major difference lies in the modeling of the fossil fuel market. One could argue that at the moment the monopolist is abandoning the market of the policy-active region, which then makes the transition to renewables, the price set by the monopolist should be continuous, due to the possibility of arbitrage. However, the assumption of perfect arbitrage leading to price continuity can be questioned, and has been questioned in the scientific literature as well as in field of policy design (cf. Hoel, 1984; Jaffe and

Soligo, 2002; Neumann and Zachmann, 2009). Nations and private actors do have built strategic oil reserves (including in tankers), waiting for (the threat of) higher prices. However, reliable estimates of such global oil inventories are lacking and therefore the importance and likelihood of arbitrage can be subject to debate (cf. Strumpf and Friedman, 2016). Van der Meijden et al. (2018, p. 154) also argue that there "is a disagreement in the literature on whether the use of the reserves is an effective tool to stabilize the oil markets and on whether the existing inventory is sufficient. Yet, these stockpiles might facilitate speculation." However, they only analyze the case of perfect arbitrage with a continuous price path in the sequel of their paper.

Here, we examine the case in which the price path set by the monopolist may be discontinuous. This might occur if the costs of storing oil are prohibitively high. We provide a full characterization of the equilibrium on the oil market in the presence of differentiated climate policies and monopolistic supply. We find equilibria with double limit pricing, i.e., equilibria where first there is an interval of time where the monopolist marginally undercuts the price of renewables in the policy-active region and later there is an interval with limit pricing in the policy-inactive region. The limit-pricing phenomenon is well known in the industrial organization literature but it is shown here that is can occur also in resource economics. For a single market this was already shown long time ago by Salant (l979), Hoel (1978) and Stiglitz and Dasgupta (1982). Here we demonstrate limit-pricing behavior in a sequence of markets. We show that the final limit-pricing phase necessarily occurs, whereas the intermediate limit-pricing phase may be degenerate. We also perform an analysis of the impact of unilateral policy changes. We find that an increase of the carbon tax or the renewables subsidy only leads to larger initial extraction if the monopolist starts with limit-pricing in the policy-active region from the beginning. If the fossil price is below the limit price in the policy-active region initially, a carbon tax and a renewables subsidy both cause a *decrease* in initial resource supply. Hence, a *weak* green paradox does occur in the former, but not in the latter case. However, the subsidy always gives causes more rapid depletion of the resource, which could lead to a *strong* green paradox.[1] The carbon tax

[1]A *weak* green paradox is said to occur if climate policies result in an increase in current carbon emissions, whereas a strong green paradox entails an increase in the present value of climate damages (cf. Gerlagh, 2011).

has an ambiguous effect on the speed of depletion.

The problem we address is not only interesting from an economic and a policy perspective, but it also raises an interesting and challenging optimal control problem, because, depending on the market on which the monopolist focuses, demand conditions differ. Hence, the analysis gives rise to a two-stage optimal control problem with a possible price jump at the transition from serving two markets to serving a single market. We solve for the entire equilibrium under rather general conditions. Technically, our analysis is related to Hoel (1984) and Wang and Zhao (2013, 2018), who study a resource monopoly with a perfect substitute that either is only suitable for some of the resource's uses (Hoel, 1984) or is subject to a capacity constraint (Wang and Zhao, 2013, 2018), leading to a discontinuity in the demand function at the price of the substitute.

The remainder of the paper is structured as follows. Section 2 describes the model. Section 3 and derives the necessary conditions for an equilibrium. Section 4 gives a full account of the resulting equilibrium, depending on the primitives of the model. Section 5 deals with policy interventions. Section 6 concludes.

2 The monopolist's problem

Energy demand comes from two regions, A and B. Energy supply consists of renewable and of fossil resources, assumed to be perfect substitutes. Renewable energy is competitively produced in both regions at a unit cost $b > 0$, whereas fossil fuel is supplied by a monopolist in a third region, where there is no domestic demand for energy. The monopolist cannot discriminate across regions. It has an initial resource stock S_0. The extraction rate is denoted by q and the constant marginal extraction cost by k.[2] Region A conducts an active climate policy by imposing a constant unit carbon tax τ on its consumers and giving them a constant unit subsidy σ on the use of renewables. With p denoting the producer price of fossil fuel, region A's consumer prices for fossil fuel and renewables are thus $p+\tau$ and $b-\sigma$, respectively. Region B is policy-inactive. Aggregate demand for fossil fuel is $q_A + q_B$, consisting of demand q_A from region A and demand

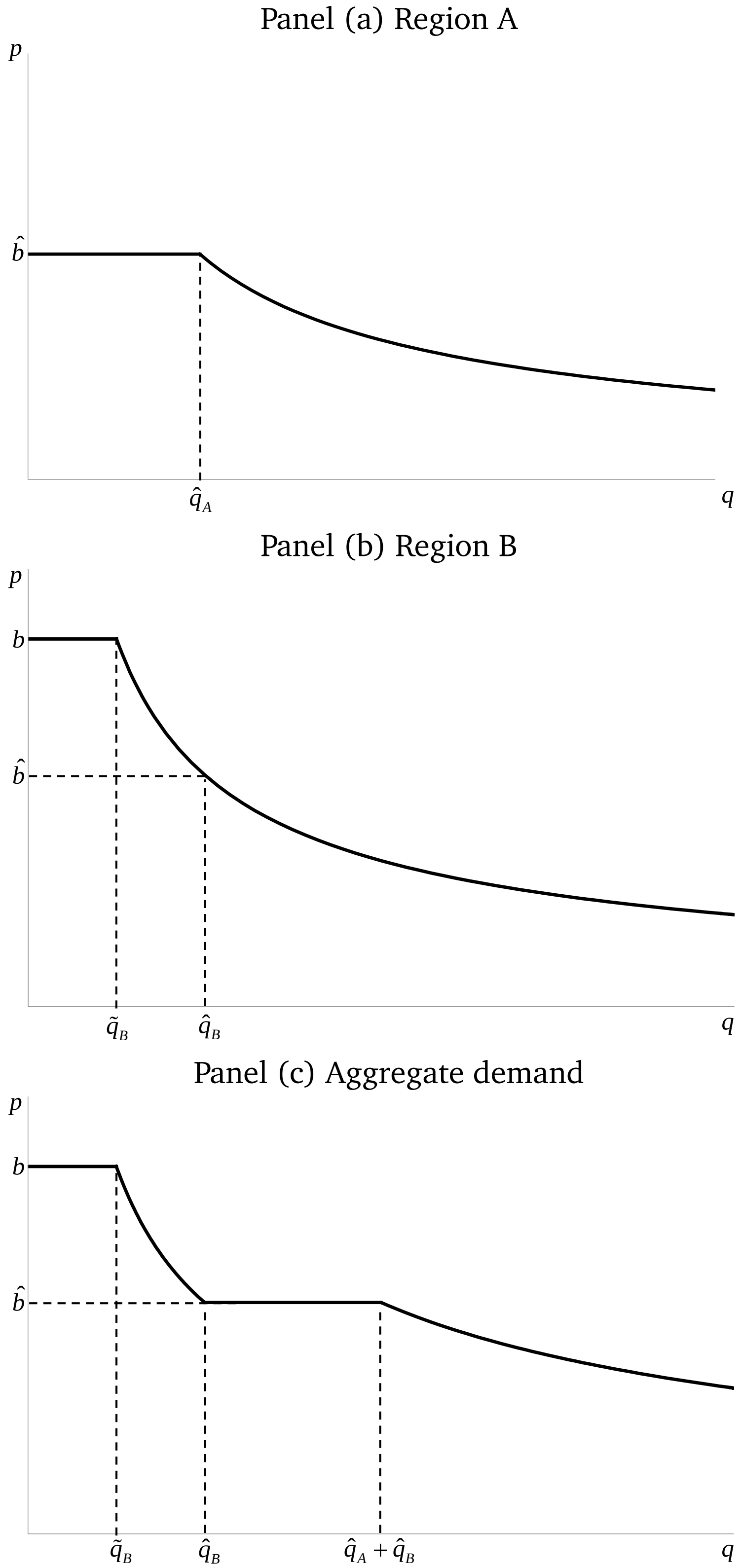
[2]We impose constant marginal extraction costs for simplicity. In the conclusion, we discuss the consequences of allowing for nonlinear costs.

q_B from region B. The monopolist faces a problem in two stages. Stage 1 starts at time zero and lasts until time T_2. During stage 1, we have $p + \tau \leq b - \sigma$. Hence, there is fossil fuel demand from region B, and possibly from region A as well. Stage 2 starts at time T_2 and lasts until time T_4. In this stage we have $p + \tau > b - \sigma$ and $p \leq b$, so there is no fossil demand anymore from region A. Time T_4 is the final moment at which extraction takes place. Both the switching time T_2 and the final time of fossil fuel use T_4 are optimally chosen by the monopolist. Each stage consists of two intermediate phases. In the first stage, from time zero until time $T_1 \leq T_2$ the consumer price of fossil fuel is strictly below the consumer price of renewables: $p + \tau < b - \sigma$, whereas from T_1 onwards there is limit pricing for region A, meaning $p + \tau = b - \sigma$. The two phases in stage 2 consist of a phase from T_2 until T_3 with $p + \tau > b - \sigma$ and $p < b$ and a second phase from T_3 until T_4 with $p = b$, with limit pricing for region B. The optimality of this sequence of phases is formally demonstrated below. We denote the producer price in the first stage, when $t < T_2$, by $p_1(q)$, and the producer price when $t \geq T_2$, by $p_2(q)$. We also define $\hat{b} \equiv b - \sigma - \tau$.

Demand is illustrated in Figure 1, where we use $\hat{q}_i$ (with $i = A, B$) to denote maximal demand for fossil fuel in region i if the consumer price is $b - \sigma$, or, equivalently, if the producer price is $\hat{b}$. Aggregate maximal demand at this price is defined as $\hat{q} \equiv \hat{q}_A + \hat{q}_B$. Similarly, $\tilde{q}_i$ represents maximal demand in region i if the consumer price is b, and we define $\tilde{q} \equiv \tilde{q}_A + \tilde{q}_B$ (of course $\tilde{q}_A = 0$ if $\hat{b} < b$). We assume $k < b$, which implies that at some instant of time all fossil fuel will be exhausted.

Instantaneous profits in stage j (with $j = 1, 2$) are denoted by $\Pi_j(q) = p_j(q)q - kq$, with $q \equiv q_A + q_B$, and are assumed to be concave for both j. We tackle the maximization problem of the monopolist by using two-stage optimal control theory (cf. Tomiyama, 1985; Makris, 2001; Valente, 2010). The idea is to first solve the problems in the two stages separately for given $T_2 \geq 0$, $T_4 \geq T_2 \geq 0$ and S_{T_2}, and to subsequently determine the optimal T_2, T_4 and S_{T_2}, where S_{T_2} denotes the remaining stock at instant of time T_2.

Figure 1: Regional and aggregate demand



3 Necessary conditions for an equilibrium

The stage 1 problem reads

$$\max_{q} \int_{0}^{T_2} e^{-rt}\Pi_1(q(t))dt,$$

subject to

$$\dot{S}(t) = -q(t), \quad q(t) \geq 0, \quad S(t) \geq 0, \quad S(0) = S_0, \quad S(T_2) = S_{T_2}, \tag{1a}$$

$$\hat{b} \geq p_1(q(t)). \tag{1b}$$

Here, $S(t)$ is the stock of fossil fuel at instant of time t and r is the constant interest rate. By $\Lambda_1(T_2, S_{T_2})$ we denote the maximal profits in the first stage.

The stage 2 problem reads

$$\max_{q} \int_{T_2}^{T_4} e^{-rt}\Pi_2(q(t))dt,$$

subject to

$$\dot{S}(t) = -q(t), \quad q(t) \geq 0, \quad S(t) \geq 0, \quad S(T_2) = S_{T_2}, \tag{2a}$$

$$b \geq p_2(q(t)), \tag{2b}$$

$$p_2(q(t)) \geq \hat{b}. \tag{2c}$$

This yields $\Lambda_2(T_2, T_4, S_{T_2})$, the maximal profits in the second stage. Subsequently, we determine the optimal T_2, T_4 and S_{T_2} by solving

$$\max_{T_2, T_4, S_{T_2}} \Lambda_1(T_2, S_{T_2}) + \Lambda_2(T_2, T_4, S_{T_2}),$$

subject to $T_2 \geq 0$, $T_4 \geq T_2 \geq 0$ and $S_0 \geq S_{T_2} \geq 0$.

Consider the problem in the first stage. Neglecting the non-negativity constraint on the extraction rate, we write the Hamiltonian and the Lagrangian as follows:

$$\mathcal{H}_1(q, \lambda_1, t) = e^{-rt}\Pi_1(q) - \lambda_1 q, \tag{3}$$

$$\mathcal{L}_1(q, \lambda_1, t) = e^{-rt}\Pi_1(q) - \lambda_1 q + \mu_{11}(\hat{b} - p_1(q)).$$

Here the multiplier μ_{11} corresponds with (1b) and λ_1 is the shadow price of the resource stock. In the absence of stock-dependent extraction costs the shadow price λ_1 is constant. In an optimum, the Lagrangian is maximized with respect to the extraction rate. We omit the time argument when there is no danger of confusion. Hence, for $q > 0$,

$$e^{-rt}\Pi_1'(q) = \lambda_1 + \mu_{11}p_1'(q). \tag{4}$$

This equation says that if the restriction $\hat{b} \geq p_1(q)$ is not binding, so that $\mu_{11} = 0$, the present value of net marginal revenues of extraction (left-hand side) equals the shadow price of the resource λ_1. If limit pricing occurs, i.e., if $p = \hat{b}$, the monopolist would want to decrease supply and thereby increase the price. The marginal cost of not being able to do this without losing demand from region A is $-\mu_{11}p_1'(\hat{q})$.

The Hamiltonian and Lagrangian of the second stage read, respectively,

$$\mathcal{H}_2(q, \lambda_2, t) = e^{-rt}\Pi_2(q) - \lambda_2 q, \tag{5}$$

$$\mathcal{L}_2(q, \lambda_2, t) = e^{-rt}\Pi_2(q) - \lambda_2 q + \mu_{21}(b - p_2(q)) + \mu_{22}(p_2(q) - \hat{b}),$$

where μ_{21} and μ_{22} are the non-negative Lagrange multipliers associated with the inequalities (2b) and (2c), respectively. The necessary condition with respect to extraction reads

$$e^{-rt}\Pi_2'(q) = \lambda_2 + \mu_{21}p_2'(q) - \mu_{22}p_2'(q). \tag{6}$$

If the second stage is non-degenerate $T_4 > T_2 \geq 0$, the Hamiltonian for the second-stage problem should equal zero at time T_4. In shorthand[3]

$$\mathcal{H}_2(T_4) = 0. \tag{7}$$

[3]This condition is obtained by noting that $\partial\Lambda_2(T_2, T_4, S(T_2))/\partial T_4 = \mathcal{H}_2(T_4)$ (cf. Theorem 3.9 in Seierstad and Sydsæter, 1987, p. 213).

In Lemma 2 below will exclude the possibility of the optimality of only having the first stage.

This completes the characterization of the equilibrium in the separate stages. To connect the two stages, assuming both are non-degenerate, note that the monopolist has to optimize not only with respect to the extraction rate and the final moment of extraction, T_4, but also with respect to the time T_2 of abandoning market A and the remaining stock at the switching time, S_{T_2}. It has been shown by Tomiyama (1985) that in an interior solution (with $0 < T_2 < T_4$) the two additional necessary conditions associated with these two decisions are given by[4]

$$\mathcal{H}_1^- = \mathcal{H}_2^+, \tag{8}$$

$$\lambda_1 = \lambda_2, \tag{9}$$

where we use the short-hand notation $x^- \equiv \lim_{t\uparrow T_2} x(t)$ and $x^+ \equiv \lim_{t\downarrow T_2} x(t)$ in (8) and in the sequel as well. Condition (9) allows us to leave out the index of the co-state variables in the remainder of the paper. The following lemma states that the price path is non-decreasing over time and is continuous within each of the two stages but is discontinuous at the transition from stage 1 to stage 2.

Lemma 1

(i) The equilibrium price path is non-decreasing over time.

(ii) The equilibrium price path is continuous for all t except at T_2, where it jumps up if $T_4 > T_2 > 0$.

(iii) $T_4 > T_3$.

Proof. See Appendix A.1. □

Intuitively, a decreasing price would imply decreasing marginal profits over time. This cannot be an equilibrium as the monopolist would then prefer early over later extraction. Furthermore, a jump in the price would imply a jump in marginal profits,

[4] Condition (8) is obtained by noting that $\partial\Lambda_1(T_2, S(T_2))/\partial T_2 = \mathcal{H}_1(T_2)$ and $\Lambda_2(T_2, T_4, S(T_2))/\partial T_2 = -\mathcal{H}_2(T_2)$ (cf. Theorem 3.9 in Seierstad and Sydsæter, 1987, p. 213).

which cannot occur along continuous parts of the marginal profit function, due to the continuity of the shadow price. However, as soon as demand from region A vanishes, the price of fossil fuel *in*creases discontinuously. The reason is that if the monopolist would decide to shift from supplying both markets to merely supplying market B without increasing the price, its instantaneous profits would decrease. Hence, it would then be better for the monopolist to keep on serving both markets. Finally, without a final limit-pricing phase, marginal profits would jump up at $T_3 = T_4$, implying that it would be profitable for the monopolist to conserve the last unit of the resource and sell it after $T_3 = T_4$, which contradicts optimality.

4 Equilibrium

The equilibrium consists of a number of consecutive extraction phases. In this section we provide a full description of all possible sequences of phases, depending on the primitives of the model. From Lemma 1 we know that there is always a final phase with limit pricing for region B. However, it could well be that other phases are degenerate. Hence, our aim is to determine the conditions under which the equilibrium takes a particular form.

4.1 Preliminaries

First, we make a distinction according to the signs of the marginal profits during the two limit-pricing phases: in the first stage evaluated in $\hat{q}$, $\Pi_1'(\hat{q})$, and in the second stage evaluated in $\tilde{q}$, $\Pi_2'(\tilde{q})$. With $\Pi_1'(\hat{q}) < 0$ strict concavity of Π_1 implies $\Pi_1^{'}(q) < 0$ for all $q > \hat{q}$. Hence, in view of (4) there is no initial phase with $q > \hat{q}$. Therefore $T_1 = 0$. Also, $\Pi_2^{'}(\tilde{q}) < 0$ implies $\Pi_2^{'}(q) < 0$ for all $q > \tilde{q}$, because of (6), and therefore $T_3 = T_2$.

Second, suppose $S_0 = \infty$. Then, we have $\lambda = 0$. Now, if $\Pi_1'(\hat{q}) < 0$ we get from (4) that $q^- = \hat{q}$. However, if $\Pi_1'(\hat{q}) \geq 0$, then q^- can be solved from $\Pi_1'(q^-) = 0$. Similarly, if $\Pi_2'(\tilde{q}) < 0$ we get from (6) that $q^+ = \tilde{q}$. However, if $\Pi_2'(\tilde{q}) \geq 0$, then q^+ can be solved from $\Pi_2'(q^+) = 0$. Let us now define

$$\bar{q} = \begin{cases} \Pi_1'^{-1}(0) & \text{if } \Pi_1'(\hat{q}) \geq 0 \\ \hat{q} & \text{if } \Pi_1'(\hat{q}) < 0 \end{cases},$$

$$\underline{q} = \begin{cases} \Pi_2'^{-1}(0) & \text{if } \Pi_2'(\tilde{q}) \geq 0 \\ \tilde{q} & \text{if } \Pi_2'(\tilde{q}) < 0 \end{cases},$$

and take an arbitrarily large initial stock, implying that $\lambda \approx 0$. According to (4) and (6), extraction in stage 1 and 2 would indeed equal $\bar{q}$ and $\underline{q}$, respectively. Then, if

$$\Pi_1(\bar{q}) - \Pi_2(\underline{q}) \leq 0,$$

we get $\mathcal{H}_1 \leq \mathcal{H}_2$ from the beginning. As a result, stage 1 is degenerate, yielding $T_2 = 0$.[5]

Finally, let us define the functions

$$f(y) = \Pi_1(\hat{q}) - \Pi_2(y) - \Pi_2'(y)(\hat{q} - y), \tag{10}$$

$$g(x) = \Pi_1(x) - \Pi_2(\tilde{q}) - \Pi_1'(x)(x - \tilde{q}). \tag{11}$$

Note that $-f(q^+)$ measures the change in *net* profits (i.e., after subtraction of the scarcity rent) at the moment the monopolist switches from stage 1 to stage 2 with limit pricing just before the switch ($q^- = \hat{q}$) but no limit pricing immediately after ($q^+ > \tilde{q}$). Similarly, $-g(q^-)$ measures the change in *net* profits at the switching instant if there is no limit pricing just before ($q^- > \hat{q}$) but there is limit-pricing immediately after the switch ($q^+ = \tilde{q}$).

Three properties of (10) and (11) will prove to be useful for characterizing the equilibrium. First, for $\hat{q}_B > y > \tilde{q}$

$$f'(y) = -\Pi_2''(y)(\hat{q} - y) > 0, \tag{12}$$

[5]It can be shown that $\dot{\mathcal{H}}_1 - \dot{\mathcal{H}}_2 = -r(\mathcal{H}_1 - \mathcal{H}_2) - r\lambda(q_1 - q_2)$, where q_i denotes the optimal q in stage $i = 1, 2$, with $q_1 > q_2$. Hence, if $\mathcal{H}_1(0) \leq \mathcal{H}_2(0)$, then $\mathcal{H}_1(t) < \mathcal{H}_2(t) \forall\ t > 0$.

so that $f(\tilde{q}) > 0$ implies $f(y) > 0$ for all $\hat{q}_B > y > \tilde{q}$. Second, we have

$$f(\hat{q}_B) = (\hat{b} - k - \Pi_2'(\hat{q}_B))(\hat{q} - \hat{q}_B) = -\hat{q}_B p_2'(\hat{q}_B)(\hat{q} - \hat{q}_B) > 0.$$

Hence, $f(\tilde{q}) < 0$ implies that $f(y) = 0$ has a solution for $\tilde{q} < y < \hat{q}_B$. Third, for $x > \hat{q}$

$$g'(x) = -\Pi_1''(x)(x - \tilde{q}) > 0, \tag{13}$$

so that $g(\hat{q}) > 0$ implies $g(x) > 0$ for all $x > \hat{q}$.

We summarize these preliminary findings in the following lemma:

Lemma 2

(i) Suppose $\Pi_1'(\hat{q}) < 0$. Then the first phase of the first stage collapses ($T_1 = 0$).

(ii) Suppose $\Pi_2'(\tilde{q}) < 0$. Then the first phase of the second stage collapses ($T_2 = T_3$).

(iii) Suppose $\Pi_1(\bar{q}) - \Pi_2(\underline{q}) \leq 0$. Then the first stage collapses ($T_1 = T_2 = 0$).

(iv) Suppose $f(\tilde{q}) > 0$. Then there is no $\tilde{q} < y < \hat{q}_B$ with $f(y) = 0$.

(v) Suppose $f(\tilde{q}) < 0$. Then $f(y) = 0$ has a solution with $\tilde{q} < y < \hat{q}_B$.

(vi) Suppose $g(\hat{q}) > 0$. Then there is no $x > \hat{q}$ with $g(x) = 0$.

4.2 A linear example

In this section, we present an example with linear demand to show that the signs of $\Pi_1'(\hat{q})$, $\Pi_2'(\tilde{q})$, $f(\tilde{q})$ and $g(\hat{q})$ can indeed be positive and negative. The demand functions for fossil fuel read

$$q_A = \begin{cases} \alpha_A - \beta_A(p + \tau) & \text{if } p + \tau \leq b - \sigma \\ 0 & \text{if } p + \tau > b - \sigma \end{cases},$$

$$q_B = \alpha_B - \beta_B p.$$

Adding the two gives

$$q = q_A + q_B = \alpha_A + \alpha_B - (\beta_A + \beta_B)p - \beta_A \tau \text{ for } p + \tau \leq b - \sigma.$$

Using $p_1(\hat{q}) + \tau = b - \sigma$ and $p_2(\tilde{q}) = b$ yields, respectively,

$$\hat{q} = \alpha_A + \alpha_B - (\beta_A + \beta_B)(b - \sigma) + \beta_B \tau,$$
$$\tilde{q} = \alpha_B - \beta_B b.$$

In order to have an interesting problem we assume throughout that $\hat{q} > 0$ and $\tilde{q} > 0$. The inverse demand functions in the two stages read

$$p_1(q) = \frac{\alpha_A + \alpha_B}{\beta_A + \beta_B} - \frac{\beta_A}{\beta_A + \beta_B}\tau - \frac{1}{\beta_A + \beta_B}q,$$
$$p_2(q) = \frac{\alpha_B}{\beta_B} - \frac{1}{\beta_B}q.$$

The profit functions are given by

$$\Pi_1(q) = \left(\frac{\alpha_A + \alpha_B}{\beta_A + \beta_B} - \frac{\beta_A}{\beta_A + \beta_B}\tau - k - \frac{1}{\beta_A + \beta_B}q\right) q,$$
$$\Pi_2(q) = \left(\frac{\alpha_B}{\beta_B} - \frac{1}{\beta_B}q - k\right) q.$$

Marginal profits at $q = \hat{q}$ in stage 1 and at $q = \tilde{q}$ in stage 2 read, respectively,

$$\Pi_1'(\hat{q}) = 2(b - \sigma) - k - \frac{\alpha_A + \alpha_B}{\beta_A + \beta_B} - \tau - \frac{\beta_B}{\beta_A + \beta_B}\tau,$$
$$\Pi_2'(\tilde{q}) = 2b - k - \frac{\alpha_B}{\beta_B}.$$

Take $\tau = \sigma = 0$ and $k = \frac{1}{2}b$. Then

$$\Pi_1'(\hat{q}) = \frac{1}{2}b + b - \frac{\alpha_A + \alpha_B}{\beta_A + \beta_B},$$
$$\Pi_2'(\tilde{q}) = \frac{1}{2}b + b - \frac{\alpha_B}{\beta_B}.$$

To get positive $\hat{q}$ and $\tilde{q}$ we then need $b - \frac{\alpha_A + \alpha_B}{\beta_A + \beta_B} < 0$ and $b - \frac{\alpha_B}{\beta_B} < 0$, respectively. Clearly, by taking $\alpha_A = \alpha_B = \alpha$ and $\beta_A = \beta_B = \beta$ and $\frac{\alpha}{\beta} > \frac{3}{2}b$ we have $\Pi_1'(\hat{q}) < 0$ and $\Pi_2'(\tilde{q}) < 0$. On the other hand $\Pi_1'(\hat{q}) > 0$ and $\Pi_2'(\tilde{q}) > 0$ if $\frac{\alpha}{\beta} < \frac{3}{2}b$. By taking $\beta_A = \beta_B = \beta$ and α_A large enough compared to α_B so that $\frac{\alpha_A + \alpha_B}{2\beta} > \frac{3}{2}b$ and $\frac{\alpha_B}{2\beta} < \frac{3}{2}b$ we find $\Pi_1'(\hat{q}) < 0$ and $\Pi_2'(\tilde{q}) > 0$. For α_A small enough compared to α_B we get the reverse.

Furthermore, we have

$$f(\tilde{q}) = (b - \sigma - \tau - k)\hat{q} - (b-k)\tilde{q} - \left(2b - k - \frac{\alpha_B}{\beta_B}\right)(\hat{q} - \tilde{q}).$$

For both τ and σ close to zero, this expression boils down to

$$f(\tilde{q}) = \left(\frac{\alpha_B}{\beta_B} - b\right)(\hat{q} - \tilde{q}) > 0.$$

For $\tau + \sigma$ close to $b - k$, we get

$$f(\tilde{q}) = \left(\frac{\alpha_B}{\beta_B} - 2b + k\right)\hat{q} - \left(\frac{\alpha_B}{\beta_B} - b\right)\tilde{q}.$$

This expression is negative if the coefficient of $\hat{q}$ is negative. Next, let us consider $g(\hat{q})$.

$$g(\hat{q}) = \left(\frac{\alpha_A + \alpha_B}{\beta_A + \beta_B} + \frac{\beta_B}{\beta_A + \beta_B}\tau - (b - \sigma)\right)(\hat{q} - \tilde{q}) - (\tau + \sigma)\tilde{q}.$$

For $\tau = 0$, the equation $g(\hat{q}) = 0$ has two solutions:

$$\begin{aligned}
\sigma &= -\frac{\alpha_A - b\beta_A}{\beta_A + \beta_B} + \frac{\sqrt{-(\alpha_A - \beta_A b)(\alpha_B - \beta_B b)}}{\beta_A + \beta_B}, \\
\sigma &= -\frac{\alpha_A - b\beta_A}{\beta_A + \beta_B} - \frac{\sqrt{-(\alpha_A - \beta_A b)(\alpha_B - \beta_B b)}}{\beta_A + \beta_B}.
\end{aligned}$$

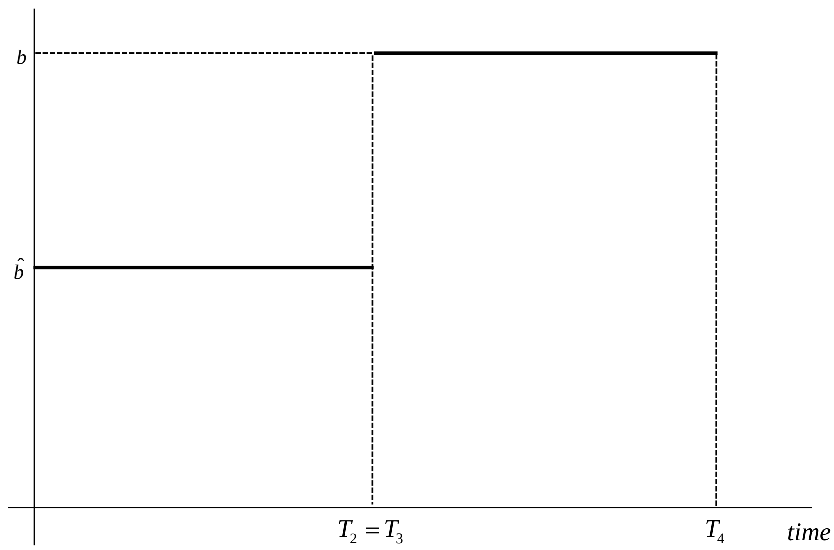
A positive $\tilde{q}$ requires $\alpha_B - \beta_B b > 0$. Hence, if $\alpha_A - \beta_A b < 0$, $g(\hat{q}) = 0$ has two real solutions of which at least one is strictly positive. Therefore, we conclude that $g(\hat{q})$ can both be positive and negative.

4.3 Characterization of the equilibrium

We now outline our strategy to characterize the equilibrium. The first step is to identify potential equilibria. The second step is to choose between them. To illustrate the approach, consider the case where $\Pi_1'(\hat{q}) < 0$ and $\Pi_2'(\tilde{q}) > 0$. Then, in qualitative terms, an equilibrium candidate is $T_4 > T_3 > T_2 > T_1 = 0$. Indeed $T_1 = 0$ because $\Pi_1'(\hat{q}) < 0$. If the equilibrium is of this type, the necessary conditions allow us to determine $T_4 - T_3$, and the price jump at T_2, both independent of the existing resource stock. Hence, we then know how much fossil fuel is used in the final period $T_4 - T_3$. We also find the amount of fossil fuel used in the period $T_3 - T_2$, since the amount extracted is known for each moment in the interval. Hence, there are two threshold levels for the initial resource stock. If the stock is small—smaller than $(T_4 - T_3)\tilde{q}$—the equilibrium only has limit pricing at the price b. We can also determine the initial stock such that $0 = T_2 < T_3$. This is the second threshold. Finally, for a still larger initial stock we have $T_2 > 0$. Hence, starting from the 'general' equilibrium, phases collapse for smaller initial resource stocks. In the sequel we sometimes omit introducing symbols for the threshold levels, if the outcomes are evident. Throughout, we will focus on equilibria in which the first stage is non-degenerate, at least not *a priori*. Hence, in view of Lemma 2 (iii), we impose $\Pi_1(\bar{q}) - \Pi_2(\underline{q}) > 0$.

4.3.1 Case 1: $\Pi_1'(\hat{q}) < 0$ and $\Pi_2'(\tilde{q}) < 0$

Figure 2: Equilibrium price path in Case 1



This case applies if demand is inelastic. In our linear example, we need a high $\frac{\alpha_A + \alpha_B}{\beta_A + \beta_B}$ or a high tax, and a high $\frac{\alpha_B}{\beta_B}$. With inelastic demand, it is to be expected that there will only be limit pricing in both stages. This is confirmed by Lemma 2 (i) and (ii). A typical equilibrium constellation is depicted in Figure 2.

The remaining question is whether $T_2 = 0$ or $T_2 > 0$. Conditions (7), (8) and (9) imply

$$\frac{\Pi_1(\hat{q}) - \Pi_2(\tilde{q})}{\hat{q} - \tilde{q}} = \frac{\Pi_2(\tilde{q})}{\tilde{q}} e^{r(T_2 - T_4)}. \tag{14}$$

We have $\frac{\Pi_1(\hat{q})}{\hat{q}} = \hat{b} - k < \frac{\Pi_2(\tilde{q})}{\tilde{q}} = b - k$. Hence,

$$\Pi_1(\hat{q}) - \Pi_2(\tilde{q}) = \frac{\Pi_2(\tilde{q})}{\tilde{q}}(\hat{q} - \tilde{q})e^{r(T_2 - T_4)} < \frac{\Pi_2(\tilde{q})}{\tilde{q}}(\hat{q} - \tilde{q}), \tag{15}$$

which gives $T_4 - T_2 > 0$. However, this does not yet imply that $T_2 > 0$. Indeed, let the threshold level S_1 be defined by

$$S_1 = (T_4 - T_2)\tilde{q}, \tag{16}$$

where $T_4 - T_2$ solves (14). Then $S_0 > S_1$ is a necessary and sufficient condition for $T_2 > 0$.

We arrive at the following proposition:

Proposition 1 *Suppose* $\Pi_1'(\hat{q}) < 0$, $\Pi_2'(\tilde{q}) < 0$ *and* $\Pi_1(\bar{q}) - \Pi_2(\underline{q}) > 0$. *If the initial stock is small, then* $T_4 > T_3 = T_2 = T_1 = 0$. *Otherwise* $T_4 > T_3 = T_2 > T_1 = 0$.

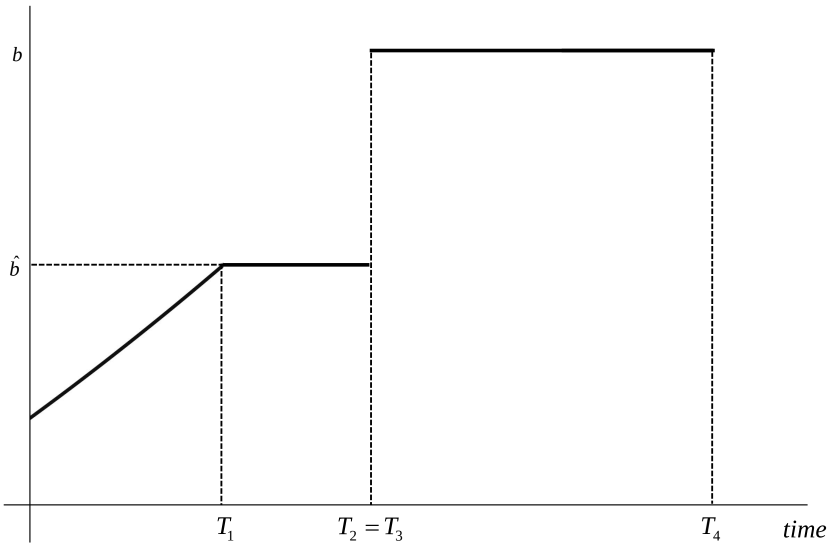
4.3.2 Case 2: $\Pi_1'(\hat{q}) > 0$ and $\Pi_2'(\tilde{q}) < 0$

The example suggests that this case is relevant for a low tax or a small $\frac{\alpha_A + \alpha_B}{\beta_A + \beta_B}$ but a large $\frac{\alpha_B}{\beta_B}$. Intuitively, with positive marginal profits during limit pricing in stage 1, the monopolist will choose for an initial phase with $p < \hat{b}$ if its initial stock is large enough. Based on Lemma 2, a typical equilibrium looks as in Figure 3, with still no interval of time where $\hat{b} < p < b$. The optimal path has a final stage, from $T_2 = T_3$ till $T_4 > T_3$ with limit pricing only, at price b. It is yet to be found out whether $T_2 > T_1 = 0$ or $T_2 > T_1 > 0$ or $T_2 = T_1 > 0$.

First, suppose that $g(\hat{q}) > 0$. In an equilibrium with $T_4 > T_2 > T_1 > 0$, conditions (4), (8), (9) require

$$e^{r(T_1 - T_2)}(\Pi_1(\hat{q}) - \Pi_2(\tilde{q})) - \Pi_1'(\hat{q})(\hat{q} - \tilde{q}) = 0, \tag{17}$$

Figure 3: Equilibrium price path in Case 2



which has a positive solution $T_2 - T_1 > 0$ if $g(\hat{q}) > 0$. From (4) and (7) we have

$$e^{-rT_1}\Pi_1'(\hat{q}) = e^{-rT_4}\frac{\Pi_2(\tilde{q})}{\tilde{q}}. \tag{18}$$

Combining (17) and (18) yields

$$e^{r(T_4-T_2)}(\Pi_1(\hat{q}) - \Pi_2(\tilde{q})) - \Pi_2(\tilde{q})\left(\frac{\hat{q}}{\tilde{q}} - 1\right) = 0. \tag{19}$$

Furthermore, we have

$$\Pi_1(\hat{q}) - \Pi_2(\tilde{q}) - \Pi_2(\tilde{q})\left(\frac{\hat{q}}{\tilde{q}} - 1\right) < \hat{q}\left(\frac{\Pi_1(\hat{q})}{\hat{q}} - \frac{\Pi_2(\tilde{q})}{\tilde{q}}\right) = \hat{q}(\hat{b} - b) < 0. \tag{20}$$

Together, (19) and (20) imply $T_4 - T_2 > 0$. In a similar way to how we defined S_1 in Case 1, our solutions for $T_2 - T_1 > 0$ and $T_4 - T_2 > 0$ now allow us to define two new threshold initial stocks (a 'small' and an 'intermediate' one) to be used in Proposition 2 below.

Now, suppose $g(\hat{q}) < 0$. The proposed equilibrium is no longer a solution: one phase must collapse so the equilibrium has $T_1 = T_2 > 0$ and $q^- > \hat{q}$. Intuitively, the change in profits due to switching from stage 1 to stage 2 only after the price has reached $\hat{b}$ is strictly positive, implying that it is profitable to make this switch earlier by skipping the limit-pricing phase of the first stage. Conditions (4), (7), (8) and (9) now imply

$$\Pi_1(q^-) - \Pi_2(\tilde{q}) - \Pi_1'(q^-)(q^- - \tilde{q}) = 0, \tag{21}$$

$$e^{r(T_4-T_1)}\Pi_1'(q^-) = \frac{\Pi_2(\tilde{q})}{\tilde{q}}, \tag{22}$$

which yields $T_4 - T_1$ and provides another threshold level.

The following proposition holds:

Proposition 2

(i) Suppose $\Pi_1'(\hat{q}) > 0$, $\Pi_2'(\tilde{q}) < 0$ *and* $g(\hat{q}) < 0$*. If the initial stock is small, then* $T_4 > T_3 = T_2 = T_1 = 0$*. If the initial stock is large, then* $T_4 > T_3 = T_2 = T_1 > 0$*.*

(ii) Suppose $\Pi_1'(\hat{q}) > 0$, $\Pi_2'(\tilde{q}) < 0$ *and* $g(\hat{q}) > 0$*. If the initial stock is small, then* $T_4 > T_3 = T_2 = T_1 = 0$*. If the initial stock takes intermediate values, then* $T_4 > T_3 = T_2 > T_1 = 0$*. If the initial stock it large, then* $T_4 > T_3 = T_2 > T_1 > 0$*.*

4.3.3 Case 3: $\Pi_1'(\hat{q}) < 0$ and $\Pi_2'(\tilde{q}) > 0$

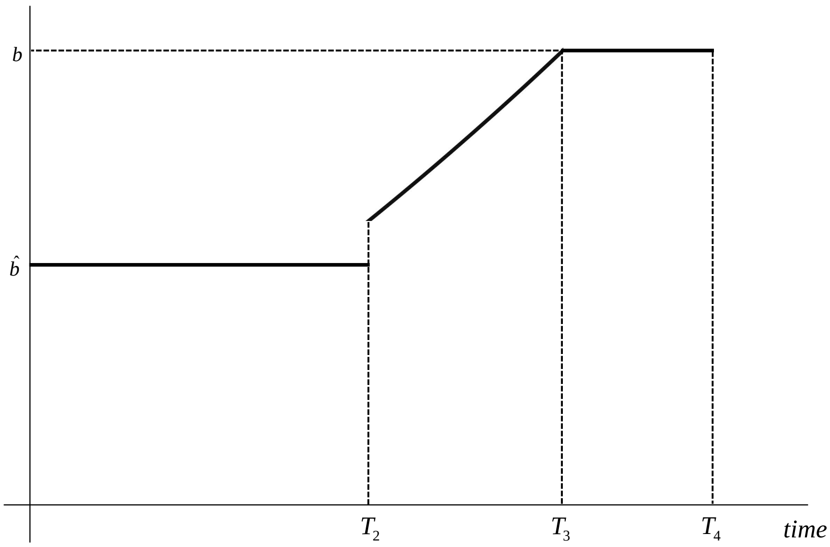
In the example, this case would occur for a high tax or a large $\frac{\alpha_A+\alpha_B}{\beta_A+\beta_B}$ but a small $\frac{\alpha_B}{\beta_B}$. As in Case 1 we have $T_1 = 0$. Intuitively, with negative marginal profits at the limit price of the first stage, the monopolist sets its price as high as possible during this stage, i.e., $p = \hat{b}$. However, because marginal profits are positive at the limit price in the second stage, the monopolist may now opt for an initial second-stage phase with a price below the limit price b. A typical equilibrium constellation is depicted in Figure 4.

First, suppose $f(\tilde{q}) < 0$. In an equilibrium with $T_4 > T_3 > T_2 > T_1 = 0$, conditions (6), (8) and (9) require $f(q^+) = 0$. According to Lemma 2 (v), this equation has a solution with $\tilde{q} < q^+ < \hat{q}_B$. Additionally, (6) and (7) imply

$$e^{r(T_4-T_3)}\Pi_2'(\tilde{q}) = \frac{\Pi_2(\tilde{q})}{\tilde{q}}, \tag{23}$$

$$e^{r(T_3-T_2)}\Pi_2'(q^+) = \Pi_2'(\tilde{q}), \tag{24}$$

Figure 4: Equilibrium price path in Case 3



which give $T_4 - T_3 > 0$ and $T_3 - T_2 > 0$ and hence provide two initial stock threshold levels.

According to Lemma 2 (iv), the proposed sequence of phases is not an equilibrium if $f(\tilde{q}) > 0$. In this case, we must have $T_3 = T_2$. This can be seen as follows. We necessarily have $T_4 > T_3$ and $T_1 = 0$. Moreover, $T_4 - T_3$ is finite and at least one phase must collapse. Intuitively, if $f(\tilde{q}) > 0$ switching from stage 1 to stage 2 with $q^- = \hat{q}$ and $q^+ > \tilde{q}$ would imply a decrease in *net* profits at the switching instant. Hence, it would be better for the monopolist to wait for a while, let the scarcity rent increase, and then switch from the limit-pricing phase in stage 1 immediately to the limit-pricing phase in stage 2, just as in Case 1. The initial threshold can now be found by solving (14).

So, we have the following proposition:

Proposition 3

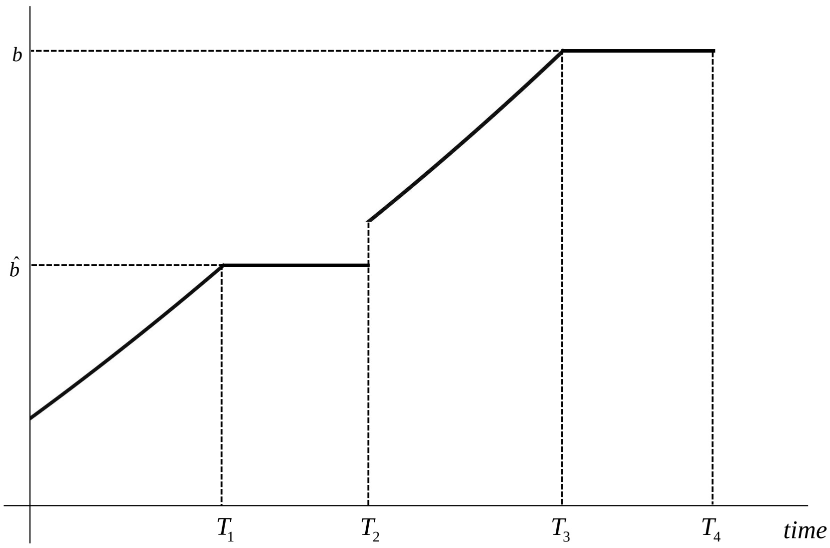
(i) Suppose $\Pi_1'(\hat{q}) < 0$, $\Pi_2'(\tilde{q}) > 0$ and $f(\tilde{q}) < 0$. If the initial stock is small, then $T_4 > T_3 = T_2 = T_1 = 0$. If the initial stock takes intermediate values, then $T_4 > T_3 > T_2 = T_1 = 0$. If the initial stock is large, then $T_4 > T_3 > T_2 > T_1 = 0$.

(ii) Suppose $\Pi_1'(\hat{q}) < 0$, $\Pi_2'(\tilde{q}) > 0$ and $f(\tilde{q}) > 0$. If the initial stock is small, then

$T_4 > T_3 = T_2 = T_1 = 0$. *If the initial stock is large, then* $T_4 > T_3 = T_2 > T_1 = 0$.

4.3.4 Case 4. Positive marginal profits $\Pi_1'(\hat{q}) > 0$ and $\Pi_2'(\tilde{q}) > 0$

Figure 5: Equilibrium price path in Case 4



Positive marginal profits occur in the example with $\alpha_A = \alpha_B,\ \beta_A = \beta_B$, small τ and $2b - k - \frac{\alpha}{\beta} > 0$. A typical equilibrium constellation is depicted in Figure 5.

First, suppose $f(\tilde{q}) < 0$. An equilibrium with $T_4 > T_3 > T_2 > T_1 > 0$ requires $f(q^+) = 0$ to have a solution with $\tilde{q} < q < \hat{q}_B$. According to Lemma 2 (v) this is indeed the case if $f(\tilde{q}) < 0$. Conditions (23) and (24) together with

$$e^{r(T_4 - T_1)}\Pi_1^{'}(\hat{q}) = \Pi_2(\tilde{q}) \tag{25}$$

yield $T_4 - T_3 > 0$, $T_3 - T_2 > 0$ and $T_2 - T_1$. Hence, they provide three initial stock threshold levels.

Now suppose $f(\tilde{q}) > 0$. Then there is no solution to $f(q^+) = 0$ with the desired properties. According to Lemma 2 (iv), the equilibrium $T_3 > T_2 > T_1$ can be excluded. Furthermore, the equilibrium $T_3 > T_2 = T_1$ can be excluded as well. The reason is that by taking the derivative of $f(y)$ with respect to $\hat{q}$ and evaluating it at q^+ we get

$$\frac{df(q^+)}{d\hat{q}} = \Pi_1^{'}(\hat{q}) - \Pi_2^{'}(q^+) > 0 \text{ for } q > \hat{q}, \tag{26}$$

where the inequality follows from $\Pi_2^{'}(q^+) = \Pi_1^{'}(q^-) < \Pi_1^{'}(\hat{q})$, due to strict concavity of Π_1 for $q > \hat{q}$. Hence, $f(\tilde{q}) > 0$ implies $T_2 = T_3$ so that the first phase of the second stage is degenerate. As in Case 3, the intuition is that with $f(\tilde{q}) > 0$ switching from stage 1 to stage 2 with $q^+ > \tilde{q}$ would imply a decrease in *net* profits at the switching instant. Therefore, it is profitable for the monopolist to wait for a while, let the scarcity rent and the resource price grow over time and immediately set the limit price after switching to the second stage.

To find out whether the second phase of the first stage exists, we need to examine the sign of $g(\hat{q})$. If $g(\hat{q}) > 0$, Lemma 2 (vi) implies $T_2 > T_1$. Hence, the equilibrium reads $T_4 > T_3 = T_2 > T_1 > 0$ for a large enough inial stock (with initial phases collapsing for smaller initial stocks). However, if $g(\hat{q}) < 0$, equation (13) implies that $g(q^-) = 0$ has a solution with $q^- > \hat{q}$, yielding $T_2 = T_1$ so that the equilibrium reads $T_4 > T_3 = T_2 = T_1 > 0$ for a large enough inial stock (again, with initial phases collapsing for smaller initial stocks). Intuitively, if $g(\hat{q}) < 0$ the change in *net* profit at the switching instant is strictly positive if the monopolist would wait until the limit price of the first stage is reached. Hence, it is profit-enhancing to make the switch earlier in time.

The following proposition summarizes the results:

Proposition 4

(i) Suppose $\Pi_1'(\hat{q}) > 0, \Pi_2'(\tilde{q}) > 0$ *and* $f(\tilde{q}) < 0$. *For a large initial resource stock the equilibrium has* $T_4 > T_3 > T_2 > T_1 > 0$. *For decreasing resource stocks the phases collapse in reverse order:* $T_j > 0$ *as long as* $T_{j-1} > 0,$ *for* $j \geq 1$.

(ii) Suppose $\Pi_1'(\hat{q}) > 0$, $\Pi_2'(\tilde{q}) > 0$, $f(\tilde{q}) > 0$ *and* $g(\hat{q}) > 0$. *For large initial resource stocks it holds that* $T_4 > T_3 = T_2 > T_1 > 0,$ *for intermediate resource stocks it holds that* $T_4 > T_3 = T_2 > T_1 = 0$ *and for small stocks it holds that* $T_4 > T_3 = T_2 = T_1 = 0$.

(iii) *Suppose* $\Pi_1'(\hat{q}) > 0$*,* $\Pi_2'(\tilde{q}) > 0$*,* $f(\tilde{q}) > 0$ *and* $g(\hat{q}) < 0$*. For large initial resource stocks it holds that* $T_4 > T_3 = T_2 = T_1 > 0$ *and for small stocks it holds that* $T_4 > T_3 = T_2 = T_1 = 0$*.*

5 Policy considerations

In the present framework several policy relevant issues can be addressed. For example, how does the introduction or tightening of climate policies in one region, while the other region stays inactive, affect the supply of fossil fuel? Of course, if the equilibrium is such that market A is not served at all, marginal changes in policies have no effect. So, we concentrate on the case where the initial producer price is not larger than $\hat{b}$. If $q(0) = \hat{q}$ then $\hat{b}$ falls so that $\hat{q}$ increases if σ or τ goes up. Hence, initial extraction goes up upon an increase in the carbon tax or an increase in the subsidy on renewables. Accordingly, both instruments cause a weak green paradox. Due to carbon leakage from region A to region B, this result differs from findings of Andrade de Sá and Daubanes (2016), which are obtained in a single-market context with inelastic demand.

However, if $q(0) > \hat{q}$ a carbon tax and a renewables subsidy both lead to a *decrease* in initial extraction: the weak green paradox disappears. This is in line with the single-market monopoly case studied in Van der Meijden and Withagen (2019). The idea is simple. The policy change reduces the profitability of the monopolist, because the constraints get tougher. Moreover, in this case (with limit pricing in the first stage after the price being below $\hat{b}$ initially) we have that total discounted profits as a function of the relevant parameters (i.e., the initial stock, the marginal cost of renewables, the tax and the subsidy) $\Lambda(S_0, b, \sigma, \tau)$ equals the Hamiltonian at time zero, divided by the discount rate r, $\mathcal{H}_1(0)/r$. This is a result well-known in standard optimal control theory. Here, in a two-stage optimal control problem, it still holds due to the continuity of the Hamiltonian at the switching instant. From the strict concavity of $(p_1(q) - k)q$ in q it follows from $\mathcal{H}_1(0) = -p_1'(q(0))q^2(0)$ that $d\mathcal{H}_1(0)/dq(0) > 0$. Therefore, we get $dq(0)/d\sigma < 0$ and $dq(0)/d\tau < 0$. Then a strengthening of climate policy in terms of a higher carbon tax or a higher subsidy on renewables reduces initial extraction and therefor mitigates climate change initially.

Although the weak green paradox does not occur in an equilibrium with $q(0) > \hat{q}$, the renewables subsidy does gives rise to earlier depletion of fossil fuel, and thereby enhances climate change and the corresponding damages. This can be seen as follows. Equation (4) implies

$$p_1'(q(0))q(0) + p_1(q(0), \tau) - k = \lambda.$$

By taking the total derivative, we get

$$d\lambda = [p''(q(0))q(0) + 2p'(q(0))]\, dq(0) - \frac{\partial p_1(q, \tau)}{\partial \tau} d\tau.$$

The term in brackets is negative, due to strict concavity of the profit function. Since $q(0)$ goes down as a consequence of the higher subsidy, as has been shown above, the shadow price λ goes up. It follows from $\mathcal{H}_2(T_4) = 0$, which implies $\lambda = (b - k)e^{-rT_4}$, that T_4 goes down. The effect of a marginally higher tax is ambiguous, however, due to the final term the total derivative above. The reason is that the tax lowers the producer price at each level of supply.

6 Conclusions

In this paper we have offered a full characterization of the equilibrium in a two-region model of resource extraction by a monopolist. The regions have different climate policies in place: one region, the 'climate club' imposes a renewables subsidy and a carbon tax, whereas the other region is policy-inactive. We focus on the case where it is too expensive for arbitrators to buy and store large reserves of the resource in order to arbitrage away jumps in the resource price. However, instantaneous arbitrage prevents the monopolist from discriminating its price between regions.

The framework gives rise to a two-stage optimal control problem for the monopolist. Two stages appear in the equilibrium: A first stage, in which both regional markets, i.e., the markets in the regions with and without climate policies, are served (at least if the initial resource stock is large enough). This first stage is followed by a second one in which only the policy-inactive region is supplied with fossil fuel. In principle there are four phases, two in each stage: one with the resource price below the limit price (and increasing over time) and one with a price at the limit price (which is constant over time). The resource price jumps *up* at the moment at which fossil demand from the climate club vanishes. The reason is that the monopolist needs an increase in the resource price to compensate for the lost demand, to prevent a downward jump in its profits at the switching instant.

Moreover, we have identified conditions under which one or more of the four phases collapse. There does always exist a final phase with limit pricing in the region without active climate policy. The intermediate limit-pricing phase at the end of the first stage, however, may be skipped by the monopolist. This occurs, for example, if marginal profits at the limit price in the first stage are positive and high compared to marginal profits in the second stage at the same price. On the contrary, if marginal profits at the limit prices of the first and the second stage are negative, the equilibrium consists of limit-pricing throughout: first at the renewables price net of the carbon tax and the renewables subsidy (with demand coming from both regions), and subsequently at the renewables price (with demand coming only from the policy-inactive region), with a positive price jump at the switching instant.

It has been shown that a *weak* green paradox only occurs if the monopolist starts with limit-pricing in the policy-active region from the beginning. If the initial price is below the limit price in the first stage, both a carbon tax and a renewables subsidy cause a decrease in initial resource supply. However, the subsidy always gives rise to earlier depletion of the resource, which could cause a *strong* green paradox. The effect of a carbon tax on the speed of depletion is ambiguous, as the tax has an immediate negative effect on resource demand for a given producer price.

Finally, it can be shown numerically that the absence of arbitrage on the fossil fuel market is beneficial to the policy-active country considering unilateral climate action, because a larger share of the resource stock is sold to this region. It is just too costly to buy and store fossil fuel resource stocks to be used in the policy-inactive region. Furthermore, the policy-inactive region in first instance benefits from policies, because of the low fossil fuel price that is due to the fact that the climate club is subsidizing renewable energy, or taxing emissions, but that later on the inactive region is confronted with much higher fossil fuel prices, once renewables take over in the policy active region.

Our study leaves some interesting issues for future research. For instance, we assume linear extraction costs to simplify the characterization of the equilibrium. Generalizing our analysis to strictly convex extraction costs does not change the potential sequences of phases in equilibrium, but it would make simultaneous use of the resource and the substitute possible. One can easily show that simultaneous use then necessarily occurs during the second part of the final limit-pricing phase and that it may occur during the second part of the intermediate limit-pricing phase. Furthermore, Appendix A.2 shows that, for a specific set of parameter values, the price jump at the switch from the first to the second stage may vanish under quadratic extraction costs. Furthermore, it would be interesting to allow for differences in climate policies between countries within the policy-active world, which would give rise to the existence of additional limit-pricing regimes. Also, deriving optimal policies in this framework could provide insights for policy makers. Other options are to allow for price discrimination between the climate club and the rest of the world, set-up costs of renewables, to consider strategic behavior on the part of the importing and exporting regions, and to examine market structures such as oligopoly or cartel-fringe (cf. Benchekroun et al., 2009, 2010, 2019). Finally, it would be interesting to allow for technological progress in the backstop technology, for R&D expenditures on developing better backstop technologies (cf. Jaakkola, 2019), and partial exhaustion if the marginal costs of the backstop technology rapidly fall below the marginal extraction cost of fossil fuels (cf. Fischer and Salant, 2017).

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A Appendix

A.1 Proof of Lemma 1

Along an interval of time with $p < \hat{b}$ we have $e^{-rt}\Pi_1'(q) = \lambda$, so that from the constancy of λ and the concavity of Π_1, the price is continuous and increasing. Along an interval of time with $b > p > \hat{b}$ we have $e^{-rt}\Pi_2'(q) = \lambda$ and the same argument applies. Obviously, the price is continuous along intervals of time where $p = \hat{b}$ or $p = b$. We now rule out a price path that has, in shorthand, $p(t_1) = \hat{b}$ and $p(t_2) < \hat{b}$ for some $t_2 > t_1 \geq 0$. It follows from (4) that if such an equilibrium path would exist

$$e^{-rt_1}\Pi_1'(q(t_1)) \leq \lambda = e^{-rt_2}\Pi_1'(q(t_2)).$$

From concavity of net revenues it follows that $q(t_1) > q(t_2)$. But $p_1(q(t_1)) > p_1(q(t_2))$ by assumption so that we get a contradiction. In the same way it can be shown that $p_2(t_1) = b$ for some $t_1 \geq 0$ implies that $p_2(t) \geq b$ for all $t > t_1$. Next, we show that the price path is continuous at T_1 where a transition takes place from $p_1 < \hat{b}$ to $p_1 = \hat{b}$. To see this, suppose an upward jump occurs at T_1. Hence $p_1(T_1^-) < p_1(T_1^+) = \hat{b}$. Then

$$e^{-rT_2}\Pi_1'(q(T_1^-)) = \lambda \geq e^{-rT_2}\Pi_1'(q(T_1^+)).$$

But this implies from concavity that $q(T_1^-) < q(T_1^+)$ contradicting $p_1(T_1^-) < p_1(T_1^+)$. The same argument holds to show that at T_3 the price is continuous.

(ii) Suppose that the price is continuous at T_2. Then $p(T_2^-) = p(T_2^+) = \hat{b}$. Since $q(T_2^-) = \hat{q}_A + \hat{q}_B$ and $q(T_2^+) = \hat{q}_B > 0$, it follows from (6), (8) and (9) that $\mathcal{H}_1(T_2) = \mathcal{H}_2(T_2) = 0$. However, substitution of (6) into (5) gives

$$\mathcal{H}_2(T_2) = -\left\{e^{-rT_2}p_2'(q(T_2^+))q(T_2^+) + \mu_{22}(T_2^+)p_2'(q(T_2^+))\right\}q(T_2^+) > 0,$$

where we have used $p_2(q(T_2^+)) = \hat{b} < b$, implying that $\mu_{21}(T_2^+) = 0$. Hence, we get a contradiction.

(iii) It cannot be optimal to have a final phase with $q(T_4) > \tilde{q}$, because then $p(T_4) < b$ and supply of fossil fuel does not meet demand. Moreover, if $q(t) > \tilde{q}$ for an interval of time (T, T_4) then $e^{-rt}\Pi_2'(q(t)) = e^{-rT_4}\frac{\Pi_2(\tilde{q})}{\tilde{q}}$ for all t close enough to T_4, which contradicts strict concavity of the profit function.

A.2 Price continuity with quadratic extraction costs

Consider the quadratic extraction cost function

$$c(q) = kq + \frac{1}{2}\omega q^2.$$

Conditions (6), (8) and (9) imply that the price is continuous at T_2 if

$$\frac{\Pi_1(\hat{q}) - \Pi_2(\hat{q}_B)}{\hat{q} - \hat{q}_B} = \Pi_2'(\hat{q}_B),$$

with $\hat{q}_B = \alpha_B - \beta_B\hat{b}$. It is straightforward to show that this equality holds if

$$\frac{\alpha_B - \beta_B(b - \sigma - \tau)}{\alpha_A - \beta_A(b - \sigma)} = \frac{1}{2}\omega\beta_B,$$

The left-hand side is nonzero, because $\hat{q}_B > 0$ by assumption. Hence, with linear extraction costs the price is discontinuous at T_2, whereas it may be continuous for other specifications, although it should be noted that in this example with quadratic costs, price continuity holds for a specific set of parameter values.