Optimal climate policy when warming rate matters

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May 13, 2020

Abstract

Studies of the Social Cost of Carbon assume climate change is a stock externality for which damages stem from warming level. However, economic and natural systems are also sensitive to the *rate at which warming occurs*. In this paper, I study the optimal carbon tax when such a feature is accounted for. I show that damages caused by warming rates do not affect optimal long-term warming, but they delay the use of the same carbon budget. Numerically, optimal carbon price should be 39% higher, compared to when damages solely stem from warming level.

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Introduction

Many human activities, in particular the burning of fossil fuels, release greenhouse gases that warm up the atmosphere and cause damage to the economy. These damages in economic analysis are typically considered to be a stock externality, driven by temperature anomaly, or the stock of atmospheric carbon dioxide, that is the "level" of climate change. However, economic and natural systems are not only sensitive to the level of change, but also to *the rate at which it occurs*, for instance because rapid changes constrain adaptation and thus induce greater damages. Accounting for this sensitivity to warming rate can affect the optimal emission pathway, and thus the Social Cost of Carbon.

There is evidence that the rate of change plays a key role in the way ecological, climate and human systems will be affected by temperature change. If ecosystems have been confronted to different climatic conditions in the past, what makes climate change so concerning is the never-seen rate at which it is occurring. More rapid rates of change limit the ability of natural systems to adapt (LoPresti et al., 2015; Hoegh-Guldberg et al., 2007; Gilman et al., 2008; Maynard et al., 2008; Malhi et al., 2009; Thackeray et al., 2010). Conversely, slower rates of change give ecosystem the time to adapt to new environmental conditions (either through behavioral or genetic changes) or to migrate in search for more favourable climates. A study suggests that for 30 percent of Earth, plant species would not be able to migrate to keep pace with projected climate change (Loarie et al., 2009). The importance of the rate of change holds in particular for systems with significant inertia, such as vegetation or soil carbon stores (Jones et al., 2009; Sihi et al., 2018). Coral reefs may also not be able to adapt to rapid rates of change (Maynard et al., 2008; Hoegh-Guldberg, 2009), because the rate of carbon absorption by the deep ocean is limited (Lenton et al., 2008).

Rapid rates of change can also contribute to trigger non-linear dynamics in the climate system, also referred to as 'tipping points' (Lenton, 2012; Levermann and Born, 2007; Steffen et al., 2018; Wieczorek et al., 2011). For instance, the stability of thermohaline circulation, as it involves water circulation flow and thus the melting rate of glacier, is sensitive to both the rate of change in atmospheric CO2 and temperature (Stocker and Schmittner, 1997; Marotzke, 1996). A rate of warming of 0.3 °C per decade sustained over a century could lead to a collapse in thermohaline circulation (O'Neill and Oppenheimer, 2004), while the same warming of 3°C reached with slower rates of change would only lead to a slowdown.

For economies too, climate damages may stem both from a changed climate and from a changing climate. For the latter, faster changes induce greater costs or less efficient adaptation (Huntingford et al., 2008; Stafford Smith et al., 2011; New et al., 2011; Smit and Wandel, 2006). Regarding decisions involving long timescales, such as urbanisation plans, tranportation or building, faster rates of change imply that infrastructures will be confronted to a larger range of climate conditions, which makes their design more difficult and construction more expensive (Hallegatte, 2009; Fankhauser and Soare, 2013). Slower rates of change also allow for more sequential decision making and to use capital more efficiently, while rapid change would force economies to retire productive capital sooner. Conversely, some of the damages may vanish once the climate has stabilized, and that economies have adapted to new climate conditions, for instance, through the use of air conditioners or changes in crop varieties or even behavioral adaptations such as changes in work hours.

This has led some scientists to argue that the rate of temperature change should be constrained to manage climate change (O'Neill and Oppenheimer, 2004; Bowerman et al., 2011; Kallbekken et al., 2009).

In the economic literature on climate change however, the role of the rate of change has not received much attention. Evironmental externalities are usually considered as either a stock or a flow externality (Farzin, 1996; Ulph and Ulph, 1994; Van Der Ploeg and Withagen, 1991), with climate change belonging to the former category. Recent analytical models of the climate and the economy all assume that damages stem from the level of warming or the stock of atmospheric carbon dioxide (Golosov et al., 2014; Gerlagh and Liski, 2018; Dietz and Venmans, 2019). DICE, the most widely used numerical Integrated Assessment Model, follows the same route, with damages caused by warming levels, leaving aside the influence of warming rates. In other numerical IAMs, such as FUND (Tol, 1996) or PAGE (Hope et al., 1993), damages depend both on level and rate of change, with the specification depending on the sector, but the authors did not analyze specifically how the combination of both types of damages affected the outcomes.

A few studies in the 1990s have compared damages from warming level and warming rates, either in a numerical IAM (Peck and Teisberg, 1994) or in an analytical model (Tahvonen, 1995; Hoel and Isaksen, 1995), suggesting that both types of damages require different optimal climate policies. However, they do not look at the case of damages being caused by a combination of level and rate of change.

In this paper, I analyze how damages caused by both warming level and warming rate affect optimal climate policy. To do so, I use an analytical model building on Dietz and Venmans (2019), where I add the feature that damages also depend on warming rate. I show that accounting for damages from warming rate do not change the long-term optimal temperature, compared to the case when damages depend solely on the warming level. However, it warrants different emission trajectories. When damages from warming rates are factored in, initial abatement is greater, but emissions decrease less rapidly. Thus, the same carbon budget is spread over time. In the central case, optimal carbon prices should be 39% higher when accounting for rate-dependent damages. This difference tends to be lower when assuming that level-damages are high.

In section 1, I present the model and derive optimal climate policy. In section 2, I explore numerically the size of the effect. Section 3 discusses implications, perspectives and concludes.

1 Model

I build upon the model in Dietz and Venmans (2019) to analyze optimal climate policy when the warming rate induces damage. This choice is motivated by their representation of the climate system, which is in line with recent results from the climate science that after a short adjustement period of ten years, the ratio of warming on cumulated emissions is independent of both time and cumulated emissions (Matthews et al., 2009; Solomon et al., 2009; Mattauch et al., 2019).

1.1 Setting

Let us assume an economy, producing Q using three inputs, capital K, labour L and emissions E. Labour and total factor productivity grow exogenously, respectively at rate n and g. Warming T caused by emissions reduces production. In addition to a classical exponential quadratic-damage function of warming levels T, I consider a symmetrical damage factor capturing that warming rate $\dot{T}$ reduces output.

$$Q = e^{n+g} f(K) exp\left(-\frac{\gamma}{2}T^2 - \frac{\alpha}{2}\dot{T}^2\right) exp\left(\Phi E - \frac{\phi}{2}E^2\right) \tag{1}$$

α and γ determine the sensitivity of economies repectively to warming level and warming rate. The case $\alpha = 0$ is the special case of economies only affected by warming levels considered in Dietz and Venmans (2019), and more generally in the climate-economy literature.

Agents derive utility from their consumption $u(c)$, and the social planer, assumed to be utilitarian, seeks to maximize the present discounted social welfare, written as follows:

$$max_{c,E} W = \int_0^\infty e^{(n-\rho)t} u(c) dt \tag{2}$$

Where ρ is the rate of pure time preference, at which future utility is discounted, and utility is isoelastic, given by:

$$u(c) = \frac{c^{1-\eta}}{1-\eta} \tag{3}$$

η is the resistance to intertemporal substitution, which drives intergenerational inequality aversion.

As discussed above, in line with recent scientific findings, I assume quasi-linearity between cumulative emissions and warming:

$$\dot{T} = \epsilon(\zeta S - T) \tag{4}$$

where ϵ is the initial pulse-adjustment timescale, and ζ reflects the Transient Climate Response to Cumulative Carbon Emissions. Thus, the damage factor reflecting sensitivity to the rate of change writes: $exp(-\frac{\alpha}{2}\dot{T}^2) = exp(-\frac{\alpha}{2}\epsilon^2(\zeta S - T)^2)$.

The part of production that is not consumed adds up to the capital stock k, but the stock also depreciates at rate δ. Thus, following the convention to write variables divided by effective labour $e^{(n+g)t}$ with a hat, capital follows the dynamical equation:

$$\dot{\hat{k}} = \hat{q} - \hat{c} - (\delta + n + g)\hat{k} \tag{5}$$

As in Dietz and Venmans (2019), it is reasonable, given the orders of magnitude at stake, to consider that the economy is on a balanced growth path with constant growth of output per capita as long the damage from warming rates has a small effect on the growth rate.

1.2 Optimal path

To determine the optimal emission pathway, we can write the Hamiltonian of the welfare maximization problem:

$$H = \frac{\hat{c}^{1-\eta}}{1-\eta} - \lambda^S E - \lambda^T \epsilon(\zeta S - T) + \lambda^{\hat{k}} \left[\hat{q}(\hat{k}, E, T) - \hat{c} - (\delta + n + g)\hat{k}\right] \tag{6}$$

Optimality conditions lead to:

$$\lambda^S = \hat{c}^{-\eta}\hat{q}(\Phi - \phi E) \tag{7}$$

$$\dot{\lambda}^S = (\rho - n + g(\eta - 1))\lambda^S - \epsilon\zeta\lambda^T - \hat{c}^{-\eta}\hat{q}\alpha\epsilon^2\zeta(\zeta S - T) \tag{8}$$

$$\dot{\lambda}^T = (\rho - n + g(\eta - 1) + \epsilon)\lambda^T - \hat{c}^{-\eta}\hat{q}(\gamma T - \alpha\epsilon^2(\zeta S - T)) \tag{9}$$

$$\hat{q}_{\hat{k}} - \delta = \eta(\frac{\dot{\hat{c}}}{\hat{c}} + g) + \rho \tag{10}$$

Integrating equation 9 gives:

$$\lambda^T = \int_t^{\infty} e^{-(\rho - n + g(\eta-1) + \epsilon)(u-t)} \hat{c}^{-\eta}\hat{q}(\gamma T - \alpha\epsilon^2(\zeta S - T))du \tag{11}$$

Given that the climate system adjusts quickly to emissions ($\epsilon \approx 0.5$), the discount rate applied to the marginal disutility of temperature change is high (around 50%). Thus, we can consider that the integral is dominated by the short-term of a few years, and over this period, $\hat{c}^{-\eta}\hat{q}(\gamma T - \alpha\epsilon^2(\zeta S - T))$ is constant:

$$\lambda^T \approx \frac{\hat{c}^{-\eta}\hat{q}(\gamma T - \alpha\epsilon^2(\zeta S - T))}{\rho - n + \epsilon + g(\eta - 1)} \tag{12}$$

Coming back to equation 8

$$\dot{\lambda}^S = (\rho - n + g(\eta - 1))\lambda^S - \epsilon\zeta\frac{\hat{c}^{-\eta}\hat{q}(\gamma T - \alpha\epsilon^2(\zeta S - T))}{\rho - n + \epsilon + g(\eta - 1)} - \hat{c}^{-\eta}\hat{q}\alpha\epsilon^2\zeta(\zeta S - T) \tag{13}$$

Deriving the equation in λ^S leads to:

$$\dot{\lambda}^S = (-\eta\frac{\dot{\tilde{c}}}{\tilde{c}} + \frac{\dot{\tilde{q}}}{\tilde{q}} - \frac{\phi\dot{E}}{\Phi - \phi E})\lambda^S \tag{14}$$

Finally, optimal conditions verify:

$$-\phi\dot{E} = \left[(\rho - n + g(\eta - 1) + \eta\frac{\dot{\hat{c}}}{\hat{c}} - \frac{\dot{\hat{q}}}{\hat{q}}\right](\Phi - \phi E) - \epsilon\zeta\frac{(\gamma T - \alpha\epsilon^2(\zeta S - T))}{\rho - n + \epsilon + g(\eta - 1)} - \alpha\epsilon^2\zeta(\zeta S - T) \tag{15}$$

The assumption of a balanced growth path leads to:

$$\dot{E} = [\rho - n + (\eta - 1)g]\,(E - \Phi/\phi) + \epsilon\frac{\zeta}{\phi}\frac{(\gamma T - \alpha\epsilon^2(\zeta S - T))}{\rho - n + \epsilon + g(\eta - 1)} + \frac{\alpha}{\phi}\epsilon^2\zeta(\zeta S - T) \tag{16}$$

The climate system adjusts quickly to CO2, so I treat the growth rate of cumulative emissions as constant in the short run, $\theta = \dot{S}/S$, and I can approximate temperature as follows:

$$T \approx \frac{\epsilon}{\epsilon + \theta}\zeta S \tag{17}$$

Substituting into the equation in $\dot{E}$, with $\zeta S - T = \frac{\theta}{\epsilon+\theta}\zeta S$, and so $\gamma T - \alpha\epsilon^2(\zeta S - T) = \frac{\epsilon}{\epsilon+\theta}(\gamma - \alpha\epsilon\theta)\zeta S$

$$\dot{E} = [\rho - n + (\eta - 1)g]\,(E - \Phi/\phi) + \frac{\zeta^2 S}{\phi}\frac{\epsilon}{\rho - n + \epsilon + g(\eta - 1)}\frac{\epsilon}{\epsilon + \theta}(\gamma - \alpha\epsilon\theta) + \frac{\alpha}{\phi}\epsilon^2\zeta^2 S\frac{\theta}{\epsilon + \theta} \tag{18}$$

$$\dot{E} = [\rho - n + (\eta - 1)g]\,(E - \Phi/\phi) + \frac{\zeta^2 S\epsilon^2}{\phi(\epsilon + \theta)}\left(\frac{\gamma - \alpha\epsilon\theta}{\rho - n + \epsilon + g(\eta - 1)} + \alpha\theta\right) \tag{19}$$

Finally, since $\dot{S} = E$, we can write:

$$\ddot{S} = [\rho - n + (\eta - 1)g]\,\dot{S} + \frac{\zeta^2\epsilon^2}{\phi(\epsilon + \theta)}\frac{\gamma + \alpha\theta(\rho - n + g(\eta - 1))}{\rho - n + \epsilon + g(\eta - 1)}S - [\rho - n + (\eta - 1)g]\,\Phi/\phi \tag{20}$$

I obtain a second-order differential equation for cumulative emissions,which can be written $\ddot{S} = a\dot{S} + bS - c$. a is the discount rate applied to the marginal damages as a proportion of output. Compared to the case of level-only damages, the only coefficient that is different is b, with $b = b_{level}(1 + \alpha\theta/\gamma(\rho - n + (\eta - 1)g)) = b_{level}(1 + a\alpha\theta/\gamma)$

In the long-term, $\theta = 0$, so it is clear that the optimal cumulative emission and optimal peak warming is unchanged compared to a case where only level damage matter $S^* = c/b = S^*_{level}$. It follows that optimal temperature levels are also identical:

$$T^* = T^*_{level} = \frac{\rho - n + \epsilon + g(\eta - 1)}{\epsilon}\frac{(\rho - n + (\eta - 1)g)\phi}{\zeta\gamma} \tag{21}$$

However, the dynamics of abatement changes when economies are also affected by warming rates. Retaining only the negative square root of the equation so the system does not diverge, cumulative emission is given by the following formula:

$$S_t = (S_0 - \frac{c}{b})exp\frac{1}{2}t\left(a - \sqrt{a^2+4b}\right) + \frac{c}{b} \tag{22}$$

Optimal emissions then write:

$$E_t = (\frac{c}{b} - S_0)\frac{1}{2}(\sqrt{a^2+4b} - a)exp\frac{1}{2}t\left(a - \sqrt{a^2+4b}\right) \tag{23}$$

In order to compare dynamics between our case and the classical case of damages depending solely on temperature level, I assume linearity between cumulative emissions and temperature in the next section.

1.3 Closed-form solution assuming no climate delay

In this section, I assume that temperature responds instantaneously to cumulative emissions, in order to obtain closed-form solutions. This simplifications rests on the fact that the climate system adjusts rapidly (within 10 years) to changes in cumulated emissions ($\epsilon = 0.5$). There is also evidence that the maximum of emissions levels is linked to the maximum warming rate (Bowerman et al., 2011), so a linear model could be an acceptable first-order representation for our purpose. $T = \zeta S$, so $\dot{T} = \zeta\dot{S} = \zeta E$.

The damage factor describing the sensitivity of production to warming rate writes: $exp(-\frac{\alpha}{2}\dot{T}^2) = exp(-\frac{\alpha}{2}\zeta^2E^2)$.

$$Q = e^{n+g}f(\hat{k})exp(-\frac{\gamma}{2}T^2 - \frac{\alpha\zeta^2+\phi}{2}E^2 + \Phi E) \tag{24}$$

The Hamiltonian of the welfare maximization problem, with this time only one state variable and two control variables is:

$$H = \frac{\hat{c}^{1-\eta}}{1-\eta} - \lambda^S E + \lambda^{\hat{k}}\left[\hat{q}(\hat{k}, E, S) - \hat{c} - (\delta+n+g)\hat{k}\right] \tag{25}$$

Optimality conditions give us:

$$\lambda^S = \hat{c}^{-\eta}\hat{q}(\Phi - (\phi+\alpha\zeta^2)E) \tag{26}$$

$$\dot{\lambda}^S = (\rho - n + g(\eta-1))\lambda^S - \hat{c}^{-\eta}\hat{q}\gamma\zeta^2 S \tag{27}$$

Derivating the expression in λ^S:

$$\dot{E} = (\rho - n + (\eta-1)g)(E - \frac{\Phi}{\phi+\alpha\zeta^2}) + \zeta^2\gamma S/(\phi+\alpha\zeta^2) \tag{28}$$

Since $\dot{S} = E$, I obtain the following differential equation for S:

$$\ddot{S} = (\rho - n + (\eta-1)g)\dot{S} + \frac{\zeta^2\gamma}{\phi+\alpha\zeta^2}S - (\rho - n + (\eta-1)g)\frac{\Phi}{\phi+\alpha\zeta^2} \tag{29}$$

Writing the equation as $\ddot{S} = a\dot{S} + bS - c$, and comparing it the case of level-only damages, we have: $a = a_{level}$, $b = b_{level}\phi/(\phi+\alpha\zeta^2)$, and $c = c_{level}\phi/(\phi+\alpha\zeta^2)$. As expected, S convergences towards the same equilibrium, the time profile of cumulative emission is given by:

$$S_t = (S_0 - c/b) exp \frac{1}{2} t(a - \sqrt{a^2 + 4b}) + c/b \tag{30}$$

Thus, emissions write:

$$E_t = (c/b - S_0) \frac{1}{2} (\sqrt{a^2 + 4b} - a) exp \frac{1}{2} t(a - \sqrt{a^2 + 4b}) \tag{31}$$

I compare emission trajectories E_t with level-only optimal emissions $E_{t,level}$:

$$\frac{E_t}{E_{t,level}} = \frac{\sqrt{a^2 + 4b} - a}{\sqrt{a^2 + 4b_{level}} - a} exp \left(\frac{1}{2} t(-\sqrt{a^2 + 4b} + \sqrt{a^2 + 4b_{level}}) \right) \tag{32}$$

Since $b < b_{level}$, initially, emissions are lower than in the level-only case. However, they decrease less rapidly, so that both emissions trajectories cross over time, and the same carbon buddget is just spread over time.

Turning to carbon price, we calculate it as the optimal marginal abatement cost for producers, which do not internalize the climate change externality:

$$p^* = Q_0 e^{(g+n)t} (\Phi - \phi E) \tag{33}$$

Thus, the opposite dynamics occurs for carbon prices. Warming rates warrant higher initial carbon prices in the short-run, and lower in the long run, compared to a case of damages only caused by warming levels.

This result comes from the fact that damages from warming reduce the rate at which marginal productivity of emissions decreases (formally equivalent to a change in ϕ), because of the flow externality they represent. However, they do not change the marginal productivity of the first emission. Alternatively, if the damage factor was an exponential-linear function of the warming rate, it would decrease in the marginal productivity of the first emission (akin to a change in Φ). Under such assumption, optimal long-term temperature would be lower.

2 Application

In this section, I propose a numerical application of the model, to evaluate the size of the effect. Assessing future level-damages is a challenging exercize, because of the diversity of impacts that climate change will induce and the many uncertainties surrounding them (Diaz and Moore, 2017; Auffhammer, 2018). The same limitation applies to the assessment of rate-dependent damages.

The difficulty is compounded because impact assessment are typically based on damages at a given temperature level, and thus do not quantify the effect of the rate of change, so that there are no values against which to calibrate the value of α, the sensitivity of economies to warming rates. To illustrate possible orders of magnitude of the parameter, I use the same strategy as in Peck and Teisberg (1994), which is to consider that a given loss of $X\%$ of production is caused by the rate of warming v.

They consider $X = 2\%$, and v is either low (0.15°C/dec), medium (0.20 °C/dec), or high (0.25°C/dec). Though this may seem somewhat arbitrary, this also allows for comparability with level-damages, because the central value γ in Dietz and Venmans (2019) is such that 2°C of warming brings about 2% loss of GDP. In the central case, a moderate warming of 2°C in the course of a century causes 2% annual loss of GDP, which vanish as temperature stabilize. I also consider a lower share of GDP loss $X = 1\%$ for each warming rate. The resulting values for α are given in table 1, range approximately from 30 to 180. Thus, I use $\alpha = 75$ as a central value, and consider 30 and 180 as a sensitivity test. All other parameters are calibrated as in Dietz and Venmans (2019).

Table 1: Value of the sensitivity to warming rates α, when considering that warming rate v causes a loss of GDP X

Warming rate (v) (deg per decade)	GDP loss (X) (in %)	α
0.15	1.00	89.34
0.20	1.00	50.25
0.25	1.00	32.16
0.15	2.00	179.58
0.20	2.00	101.01
0.25	2.00	64.65

First, I compute the evolution of the carbon price (see figure 1). When rate-damages are accounted for in the central case of $\gamma = 0.01$ and $\alpha = 75$ initial carbon prices are 39%, and the range of value I explore for α leads to increases of between 19% and 64%. However, the share of the carbon price that is due to rate-damages decreases over time. This is due to the fact that while damages from warming levels add up as we explore higher temperature levels, the externality associated with the rate of change is rather a flow externality, so that marginal emission cause the same loss, expressed as a percent of production, at every temperature levels.

Another interesting feature is that the difference between the case of level-only damage and the case of both level and rate-dependent damages is greater for low values of sensitivity to warming level (low γ). If one assumes that the sensitivity to warming level is important, then adding the effect of warming rate does not affect significantly carbon prices. For instance, for $\gamma = 0.02$, accounting for sensitivity to warming rates only increases carbon prices by 6 to 22%. Indeed, if level-damages are important, then there is a strong benefit of limiting emissions in the short-run, which also limits the rate of warming in the short-run. On the other hand, if level-damages are rather low, it is optimal to reach greater levels of warming: the effect of level-damages is moderate in the short-run, and it becomes crucial to delay the warming. In such case, carbon prices are between 45 and 148% higher.

When analyzing optimal emissions, I find the sensitivity to warming rates significantly affect optimal emissions (see figure 2). First, compared to level-only damages, initial emissions are between 16% and 52% lower in the central case of $\gamma = 0.01$. Unsurprisingly, the greater the α, the lower the initial optimal emissions. Besides,

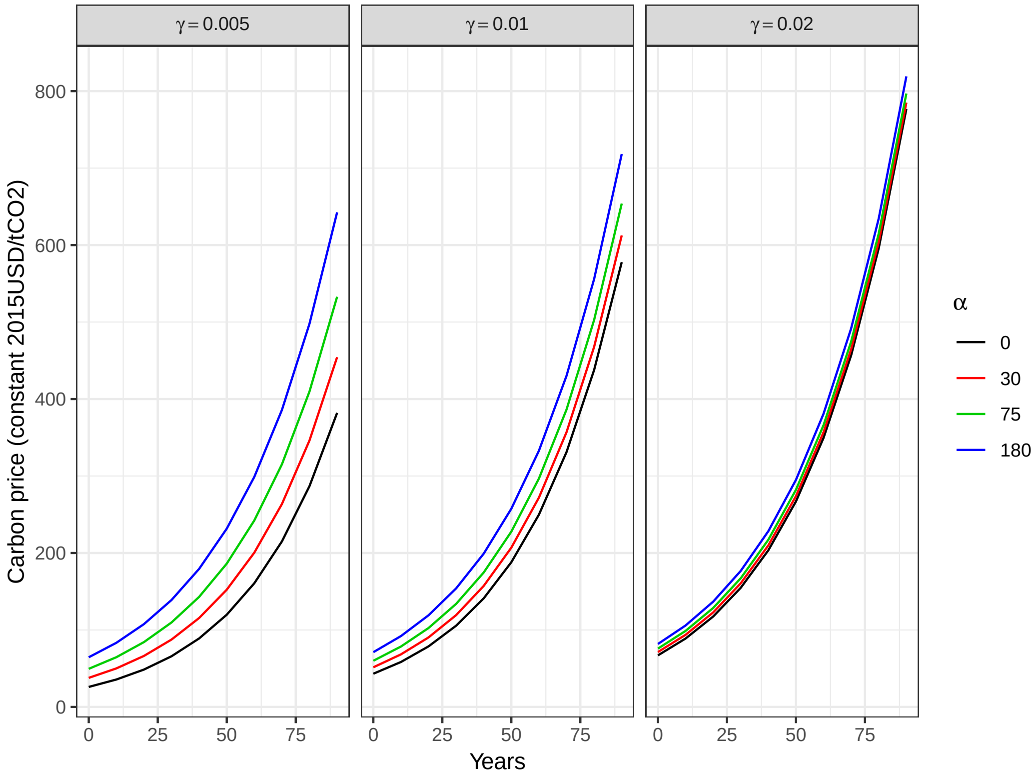

Figure 1: Evolution of carbon prices for different values calibration of parameters defining the strength of damages from warming level (γ), and warming rate (α)

emissions decrease less rapidly in the rate-damage case so that the emission curves cross the level-damage case in the long-term. This moment occurs sooner for high level-damage ($\gamma = 0.02$), between 150 and 200 years, while it occurs over a longer timeframe for lower γ.

3 Perspective and conclusion

In this article, I argue that the rate of warming plays an important role in assessing damages from climate change. I review the literature to show that both natural and economic systems have limited ability to adapt to rapid changes, thus suggesting that damages depend not only on warming levels, but also on warming rates.

Using an analytical model of the climate and the economy, I show that the rate of change does not affect optimal long-term temperature change, compared to a case when damages only depend on warming level. This comes from the fact that the marginal productivity of the first emission is unchanged under exponential-quadratic damages from the rate of warming. Sensitivity to warming rate does affect the optimal emission pathway: it is optimal to start at lower emission levels, but to reduce these emissions slowlier. Thus, the same carbon budget is spread over time. Numerically,

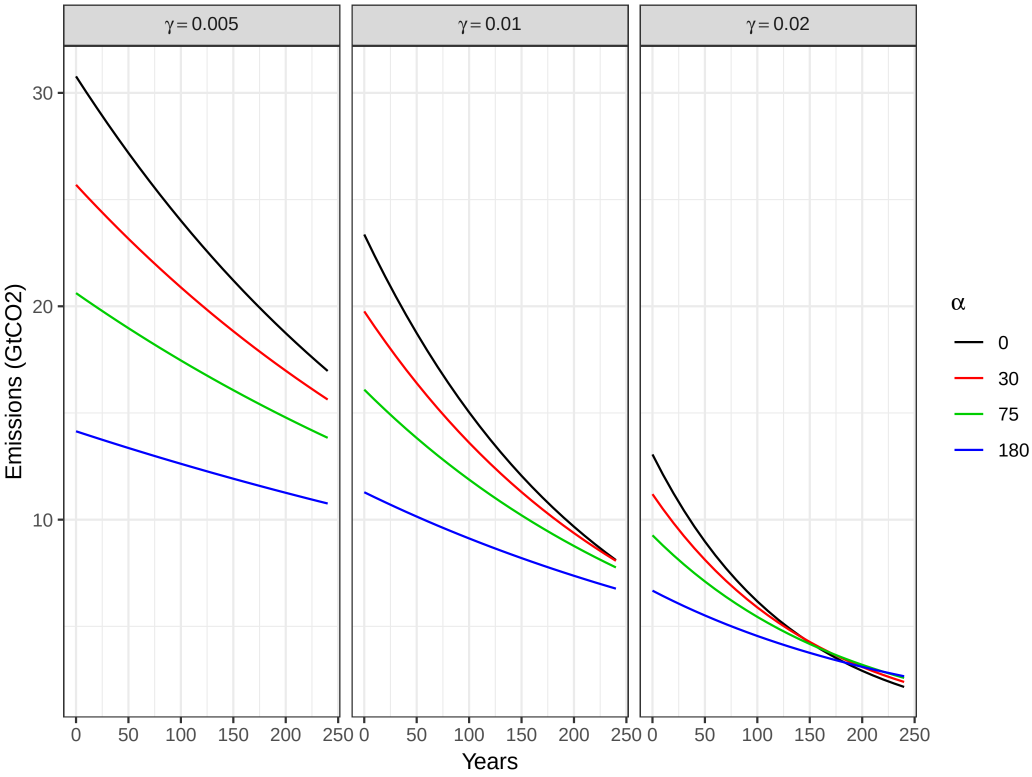

Figure 2: Evolution of emissions for different values calibration of parameters defining the strength of damages from warming levels (γ), and warming rate (α)

I estimate that in the central case, optimal carbon price should be 39% higher than when damages stem solely from warming level. Higher increases in the SCC occur for lower damages from warming rates.

This suggests that the impact of damages from warming rate on optimal emissions and carbon price could be significant. Thus, this opens up research avenues to further refine the representation of how different warming rates affect economic and natural systems. I acknowledge that both the functional form of damages and the calibration I use is questionable. For instance, as stated above, assuming that the damage factor from warming rates is exponential-linear, rather than exponential-quadratic, reduces optimal long-term warming. Besides, I assume that damages from warming rate and warming level are multiplicatively separable, while more complex interaction between both types of damages could arguably be envisaged.

Finally, accounting for the sensitivity of economies to warming rate has crucial implications for other climate policy questions, which the simplicity of the model I use does not allow me to deal with. First, the possibility to rely on negative emissions in the future raises the question of assessing overshoot temperature trajectories, in which Earth warms up to a peak before decreasing significantly (Bowerman et al., 2011). Overshoot trajectories have a very different temperature dynamics, in particular with a strong rate of temperature change in the short-term. Thus, accounting for damages from warming rates would probably affect the evaluation of such pathways. Second, given that different greenhouse gas have different lifetimes in the atmosphere, rate-dependent damages can change the trade-off across greenhouse gas (Manne and Richels, 2001), and would provide a stronger case for abating short-lived atmospheric components in the near-term, in order to slow warming rates.

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